The Physics of Sports and Excel Spreadsheets

by

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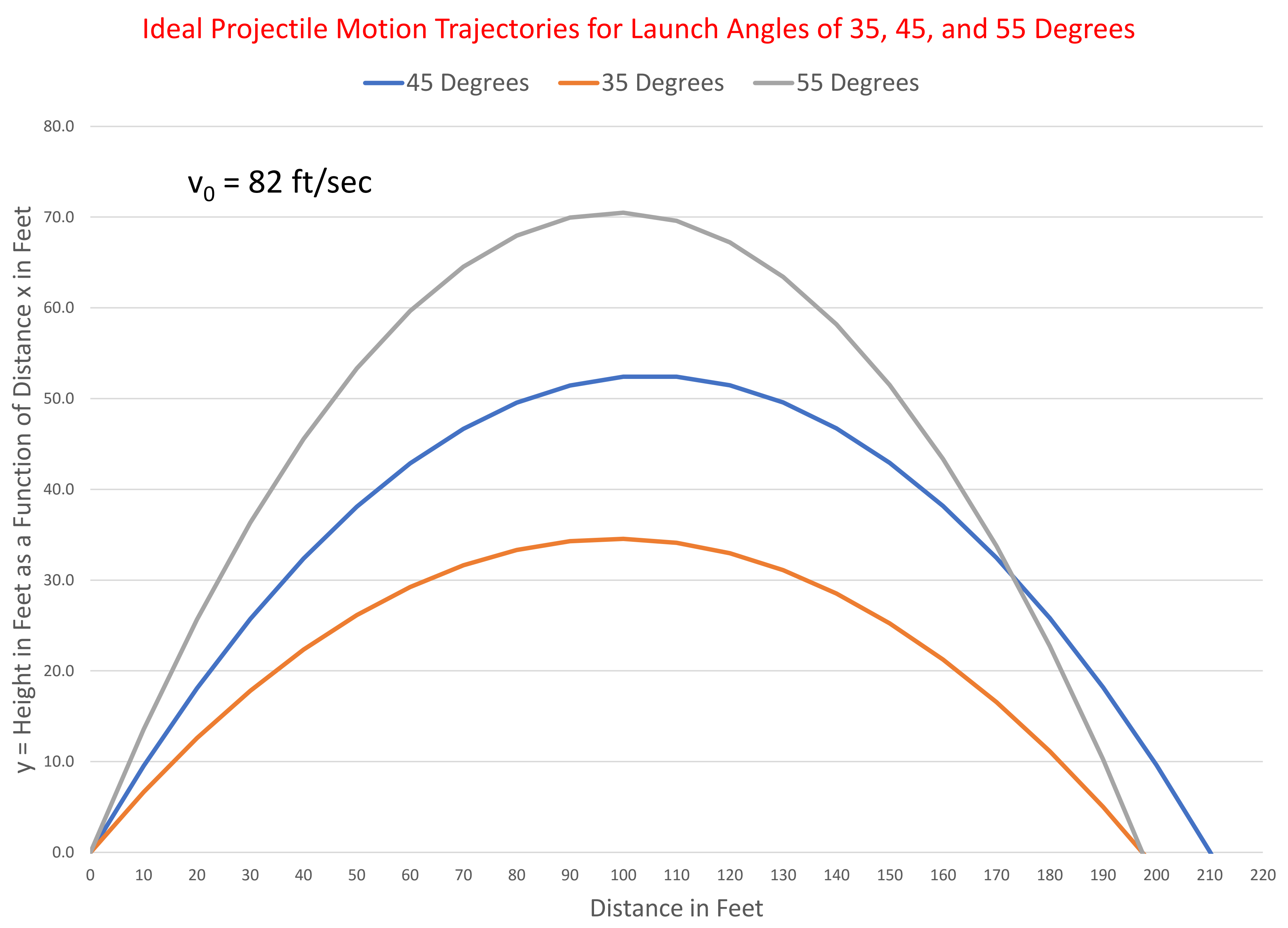
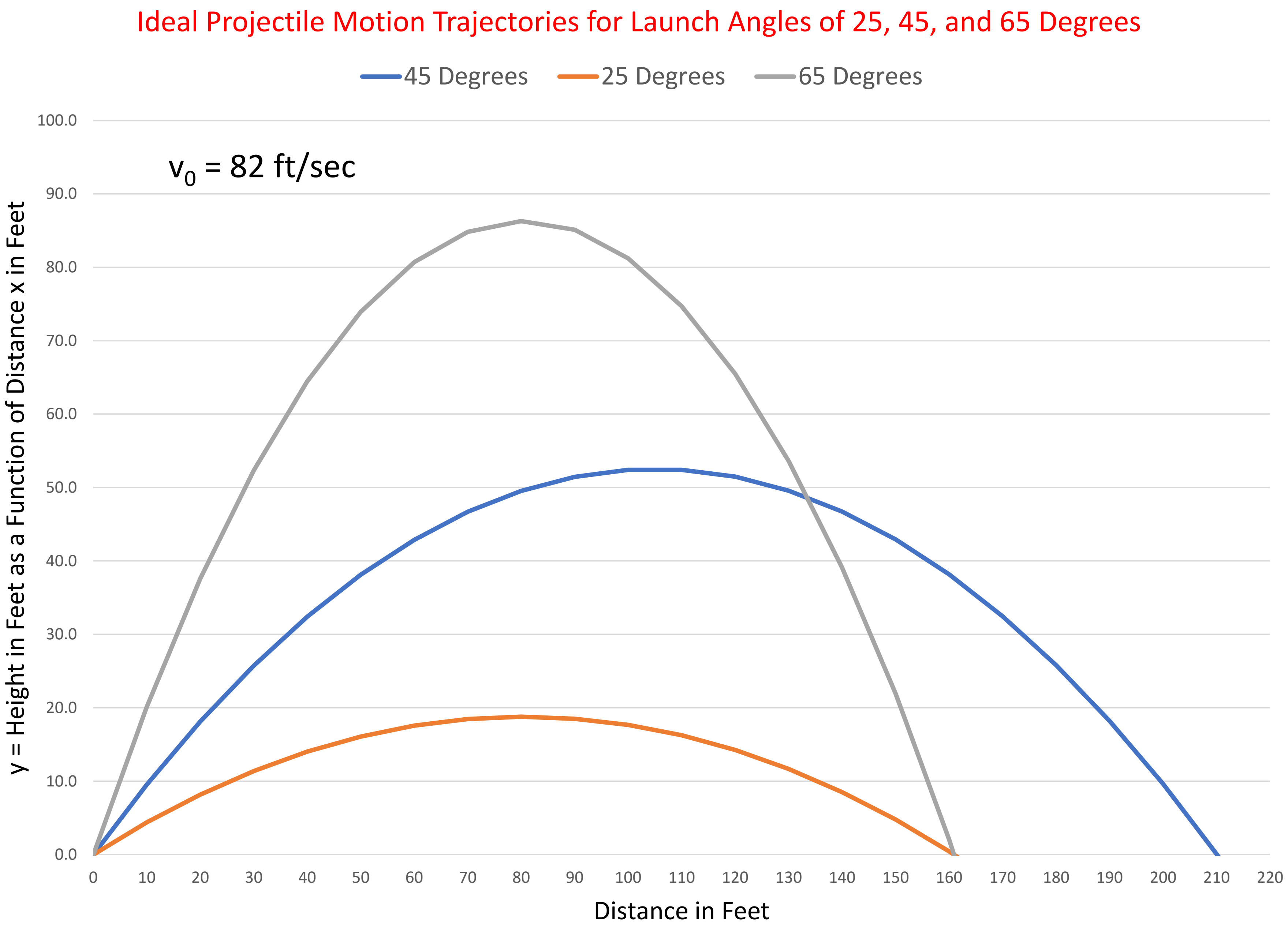
Ideal Projectile Motion - I

The Exact Solution in 2-D:

- $a_x = 0$; $a_y = -g$
- $v_x = \text{constant} = v_{0x}$
- $x = v_{0x}t$
- $v_y = v_{0y} - gt$
- $y = v_{0y}t - \frac{1}{2}gt^2$
- $y = x\tan\theta_0 - gx^2/(2v_0^2\cos^2\theta)$



Ideal Projectile Motion Trajectories for Launch Angles of 35, 45, and 55 Degrees
45 Degrees
35 Degrees
55 Degrees
v_0 = 82 ft/sec
y = Height in Feet as a Function of Distance x in Feet
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220
Distance in Feet


Ideal Projectile Motion Trajectories for Launch Angles of 25, 45, and 65 Degrees
45 Degrees
25 Degrees
65 Degrees
v_0 = 82 ft/sec
y = Height in Feet as a Function of Distance x in Feet
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220
Distance in Feet

Ideal Projectile Motion - II

The Numerical Solution in 2-D

- Using *difference* equations, we create a spreadsheet in which a motion lasting say two seconds is broken into 200 segments, each one of duration $\Delta t = 0.01$ seconds.
- Let n = the index number of the segment beginning with segment 1 from times $t = 0$ to $t = \Delta t$.
- Let $x(n)$, $y(n)$, $v_x(n)$ and $v_y(n)$ be the values at the end of the nth segment. These values will be found on the n^{th} line (i.e., row) of the spreadsheet.

Numerical Integration by Excel

- The initial values at the end of segment 1 [during the first Δt] are $x(1)$, $y(1)$, $v_x(1)$ and $v_y(1)$. For ideal projectile motion from the origin of the x-y axes, $x(1) = y(1) = 0$; and $v_{x0} = v_0\cos\theta_0$, and $v_{y0} = v_0\sin\theta_0$.

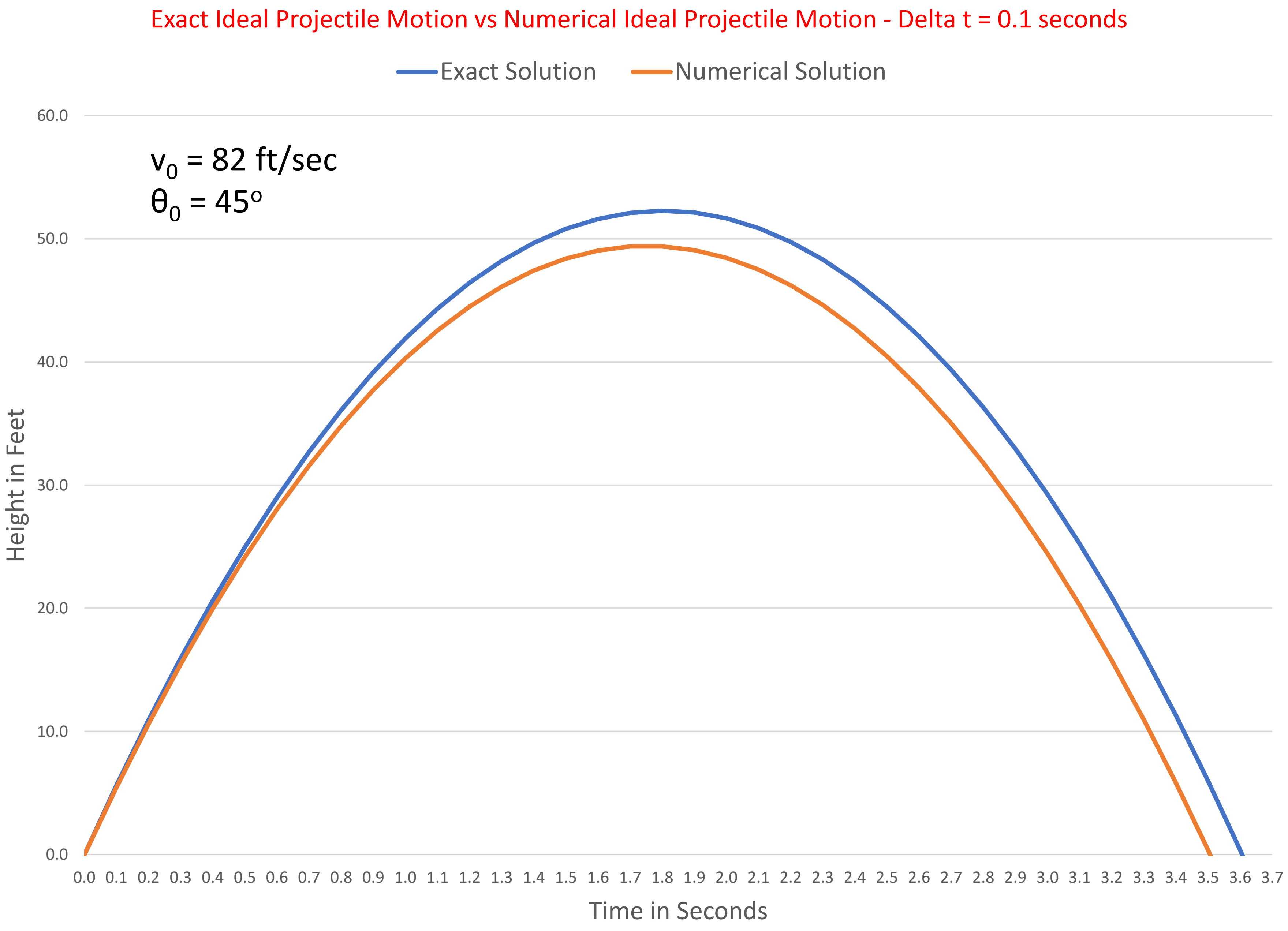
$$\text{Then: } v_x(n+1) = v_x(n) + a_x(n)\Delta t; \qquad (1)$$

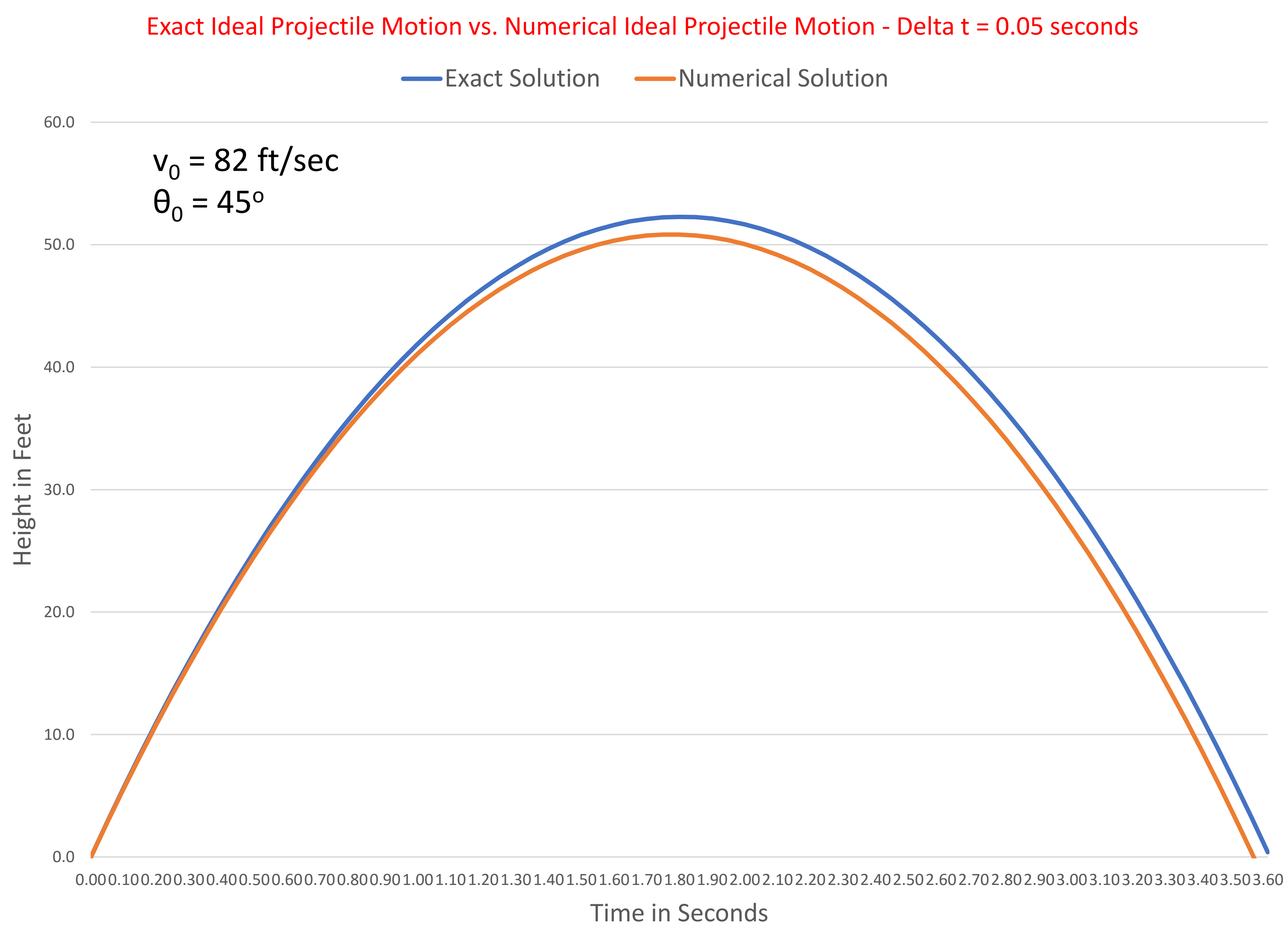
$$\text{And: } x(n+1) = x(n) + v_x(n)\Delta t + \tfrac{1}{2}a_x(n))(\Delta t)^2. \qquad (2)$$

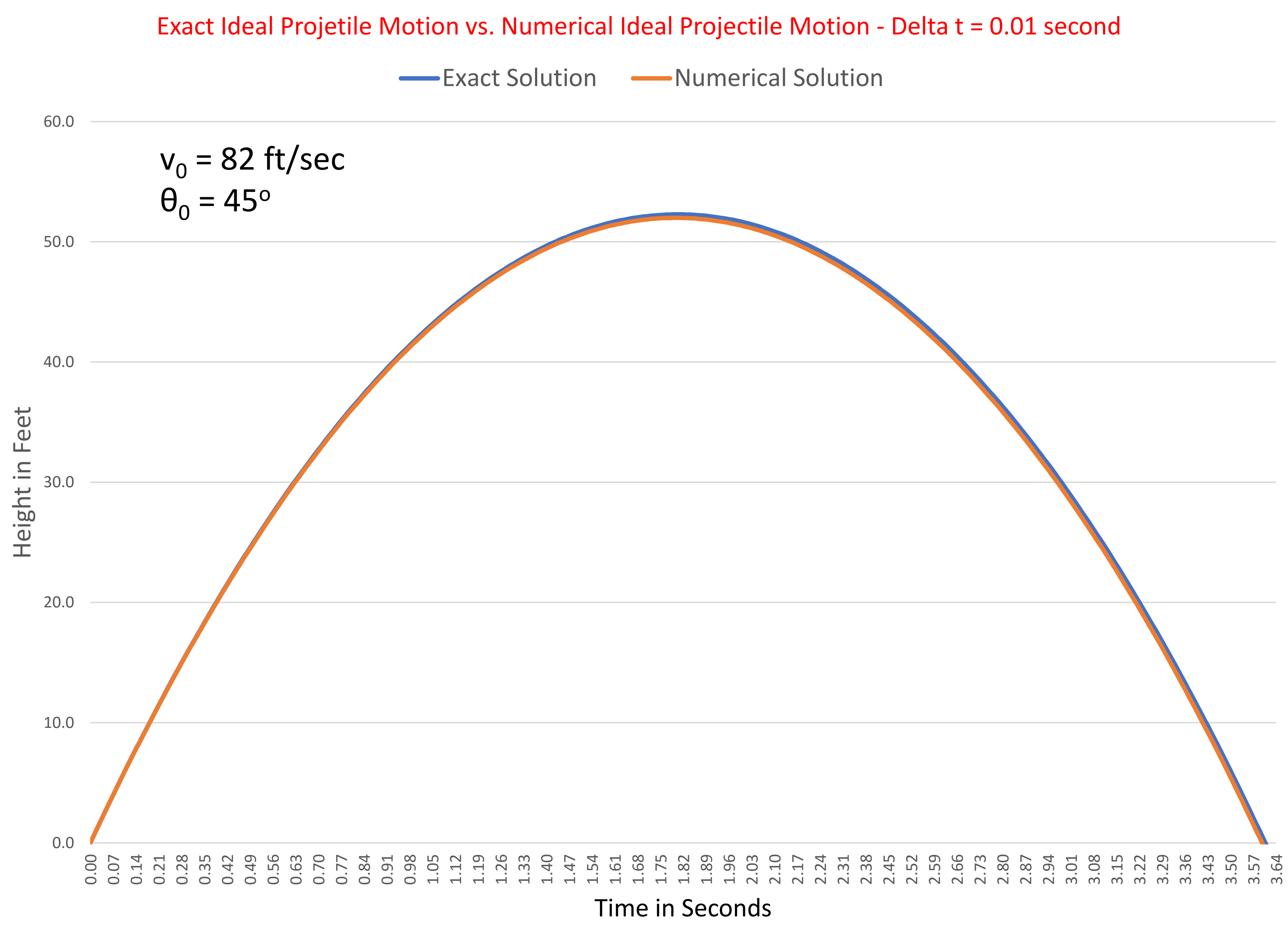
$$\text{Also: } v_y(n+1) = v_y(n) + a_y(n)\Delta t; \qquad (3)$$

$$\text{And: } y(n+1) = y(n) + v_y(n)\Delta t + \tfrac{1}{2}a_y(n)(\Delta t)^2. \qquad (4)$$

- Note the "Bootstrap;" The values in row n+1 are based on the values in row n, the previous row of the spreadsheet, which are all known!


Exact Ideal Projectile Motion vs Numerical Ideal Projectile Motion - Delta t = 0.1 seconds
Exact Solution
Numerical Solution
v_0 = 82 ft/sec
θ_0 = 45°
Height in Feet
Time in Seconds
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7


Exact Ideal Projectile Motion vs. Numerical Ideal Projectile Motion - Delta t = 0.05 seconds
Exact Solution
Numerical Solution
v_0 = 82 ft/sec
θ_0 = 45°
Height in Feet
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.00 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00 2.10 2.20 2.30 2.40 2.50 2.60 2.70 2.80 2.90 3.00 3.10 3.20 3.30 3.40 3.50 3.60
Time in Seconds


Exact Ideal Projetile Motion vs. Numerical Ideal Projectile Motion - Delta t = 0.01 second
Exact Solution
Numerical Solution
v_0 = 82 ft/sec
θ_0 = 45°
Height in Feet
Time in Seconds
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.00 0.07 0.14 0.21 0.28 0.35 0.42 0.49 0.56 0.63 0.70 0.77 0.84 0.91 0.98 1.05 1.12 1.19 1.26 1.33 1.40 1.47 1.54 1.61 1.68 1.75 1.82 1.89 1.96 2.03 2.10 2.17 2.24 2.31 2.38 2.45 2.52 2.59 2.66 2.73 2.80 2.87 2.94 3.01 3.08 3.15 3.22 3.29 3.36 3.43 3.50 3.57 3.64

Exact Solutions vs. Numerical Solutions

- When equations can be integrated exactly, those solutions work for all initial conditions. An exact solution solves a wide range of problems with slightly different, or even greatly different, initial conditions. For that reason, exact solutions are extremely powerful.
- On the other hand, numerical solutions solve one problem at a time. And for every new set of initial conditions, one must insert the new initial conditions into the first line of the spreadsheet to bootstrap the new solution uniquely determined by those new initial conditions.
- Numerical solutions are available even when exact solutions are not.

Exact Shuttlecock Motion in One Dimension - I

Free Fall from Rest (Exact solution)

$$F_{res} = -kmv^2 \qquad (1)$$

$$mg - kmv^2 = ma \qquad (2)$$

At terminal velocity, $a = 0$ and $v = V_T$ = constant, and

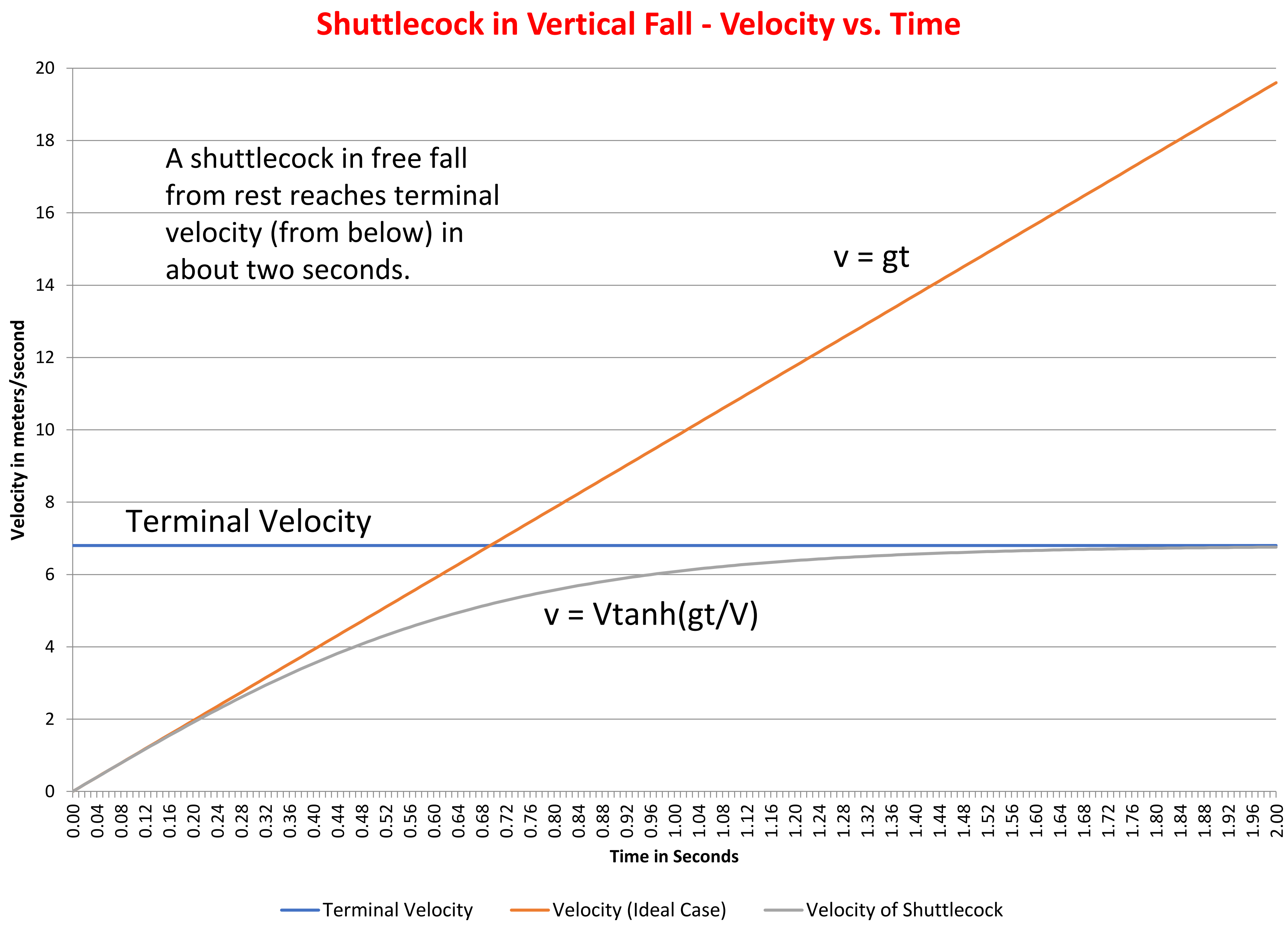
$$V_T^2 = g/k \qquad (3)$$

We learned by direct experiment that the terminal speed

$$V_T = 6.80 \text{ m/s } (15 \text{ mph}) \qquad (4)$$

The exact solution for $v(t)$ from Eq.(2) is $v(t) = V_T \tanh(gt/V_T)$ (5)

The exact solution for $y(t)$ from Eq.(5) is $y(t) = (V_T^2/g)\ln[\cosh(gt/V)]$ (6)

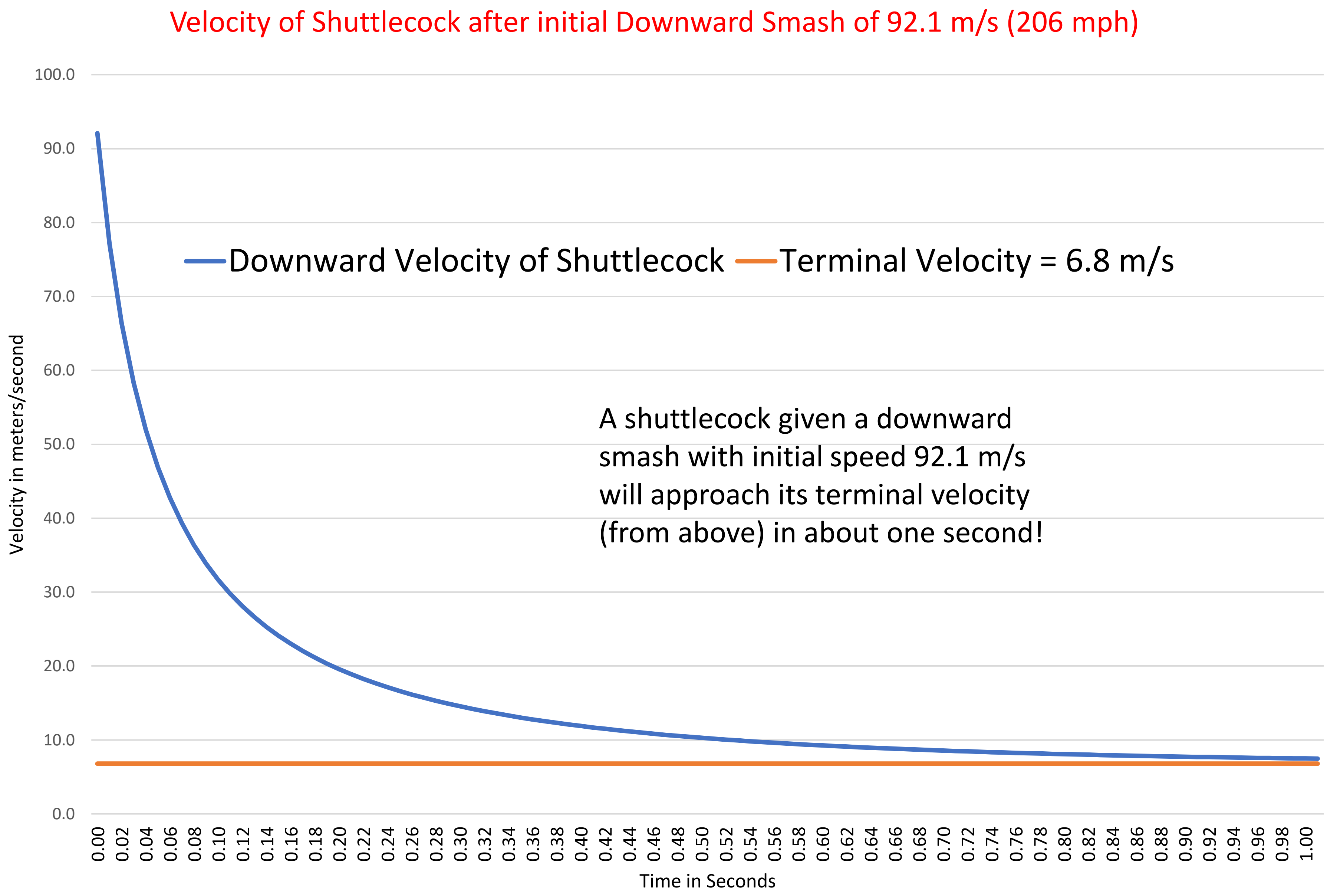

Shuttlecock in Vertical Fall - Velocity vs. Time
A shuttlecock in free fall from rest reaches terminal velocity (from below) in about two seconds.
v = gt
Terminal Velocity
v = Vtanh(gt/V)
Velocity in meters/second
Time in Seconds
Terminal Velocity
Velocity (Ideal Case)
Velocity of Shuttlecock

Exact Shuttlecock Motion in One Dimension - II

The Downward Smash

- The 2006 World Record for the highest initial speed of a shuttlecock—after being struck with a badminton racket—is 92.1 m/s (206 mph!)
- The initial speeds of well-struck shuttlecocks can approach similar speeds during serious badminton competition, which raises questions about how such high speeds are dealt with on an official badminton court that is only 20 feet wide and 44 feet long.
- For a shuttlecock launched, not from rest, but with an initial downward speed of $v_0 = 92.1$ m/s, the Eq. 1 type resistive force remains the same but the equation of motion differs from Eq. (5) because the initial conditions are so different. According to theory, the exact downward speed is given by:

$$v(t) = V\,[(v_0 + V\tanh(gt/V))/(V + v_0\tanh(gt/V))] \qquad (7)$$



Velocity of Shuttlecock after initial Downward Smash of 92.1 m/s (206 mph)
Downward Velocity of Shuttlecock
Terminal Velocity = 6.8 m/s
A shuttlecock given a downward smash with initial speed 92.1 m/s will approach its terminal velocity (from above) in about one second!
Velocity in meters/second
Time in Seconds
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

Shuttlecock Motion in Two Dimensions - I

$$\mathbf{F} = m\mathbf{a} = m(\Delta\mathbf{v}/\Delta t) \qquad (8)$$

$$F_x = m(\Delta v_x/\Delta t) \qquad (9)$$

$$F_y = m(\Delta v_y/\Delta t) \qquad (10)$$

We will assume the following resistive force on the shuttlecock:

$$\mathbf{F}_{res} = -mkv^2(\mathbf{v}/v) \qquad (11)$$

“m” is the mass of the shuttlecock, “k” is a proportionality constant, and ($\mathbf{v}/v$) is a unit vector which points in the direction of the vector velocity $\mathbf{v}$ of the shuttlecock.

Shuttlecock Motion in Two Dimensions - II

- The combination of a minus sign with the unit vector ($\mathbf{v}/v$) in Equation (11) means the force of air resistance always remains proportional to the square of the speed (v^2) but is always opposite in direction to the instantaneous velocity vector of the shuttlecock.
- Assume that the shuttlecock is launched from the origin (0,0) with initial speed v_0 and initial launch angle θ_0. We immediately find that

$$v_{x0} = v_0\cos\theta_o; \ \text{and } v_{y0} = v_0\sin\theta_0 \qquad (12)$$

Shuttlecock Motion in Two Dimensions - III

From Equations (9), (10) and (11) we find that:

$$F_x = m(\Delta v_x/\Delta t) = -mkv^2(v_x/v), \quad \text{and}$$

$$F_y = m(\Delta v_y/\Delta t) = -mg - mkv^2(v_y/v).$$

After cancelling out the mass from both sides, we find:

$$\Delta v_x = -kvv_x\Delta t \qquad (13)$$

$$\Delta v_y = -(g + kvv_y)\Delta t \qquad (14)$$

- Recall that in 2-D $\quad v^2 = v_x^2 + v_y^2 \qquad (15)$

Shuttlecock Motion in Two Dimensions - IV

- Due to the presence of v in Equations (13) and (14), they are said to be "coupled equations." That is, the equation of motion for v_x also includes v_y; the equation of motion for v_y also includes v_x; and in both cases that is due to the fact that from Eq.(15), v has both v_x and v_y components.
- Eqs. (13) and (14) cannot be solved exactly because of this coupling. The solution for velocity v_x in the x-direction <u>depends</u> on velocity v_y in the y- direction; and the solution for v_y in the y-direction <u>depends</u> on v_x in the x direction!
- This presents a major problem in terms of finding "exact solutions" for Equations (13) and (14). But it presents no problem at all in terms of using numerical integration via Excel spreadsheets!
- Recall how the numerical integration results compared favorably with the exact solutions for ideal projectile motion, provided only that the time increment Δt is made sufficiently small.

The Bootstrap Method of Numerical Integration

- Spreadsheets allow the numerical integration of Equations (13) and (14) despite the coupling. Here's why:
- We know the values of v_x and v_y at $t = 0$. They are given in Equation (11). That means we also know the value of v at $t = 0$. We simply put the values from Eq. (11) into Eq. (15) to find v.
- The zeroth row of our Excel spreadsheet will list the various values and parameters at time $t = 0$.
- Some of the column headings will carry titles such as these:
- Δt; t; v_x; v_y; v; Δv_x; Δv_y
- The entries in the zeroth row of the spreadsheet for the above 7 headings will carry the following values:
- 0.01; 0; v_{x0}; v_{y0}; 0; Δv_{x0}; Δv_{yo}

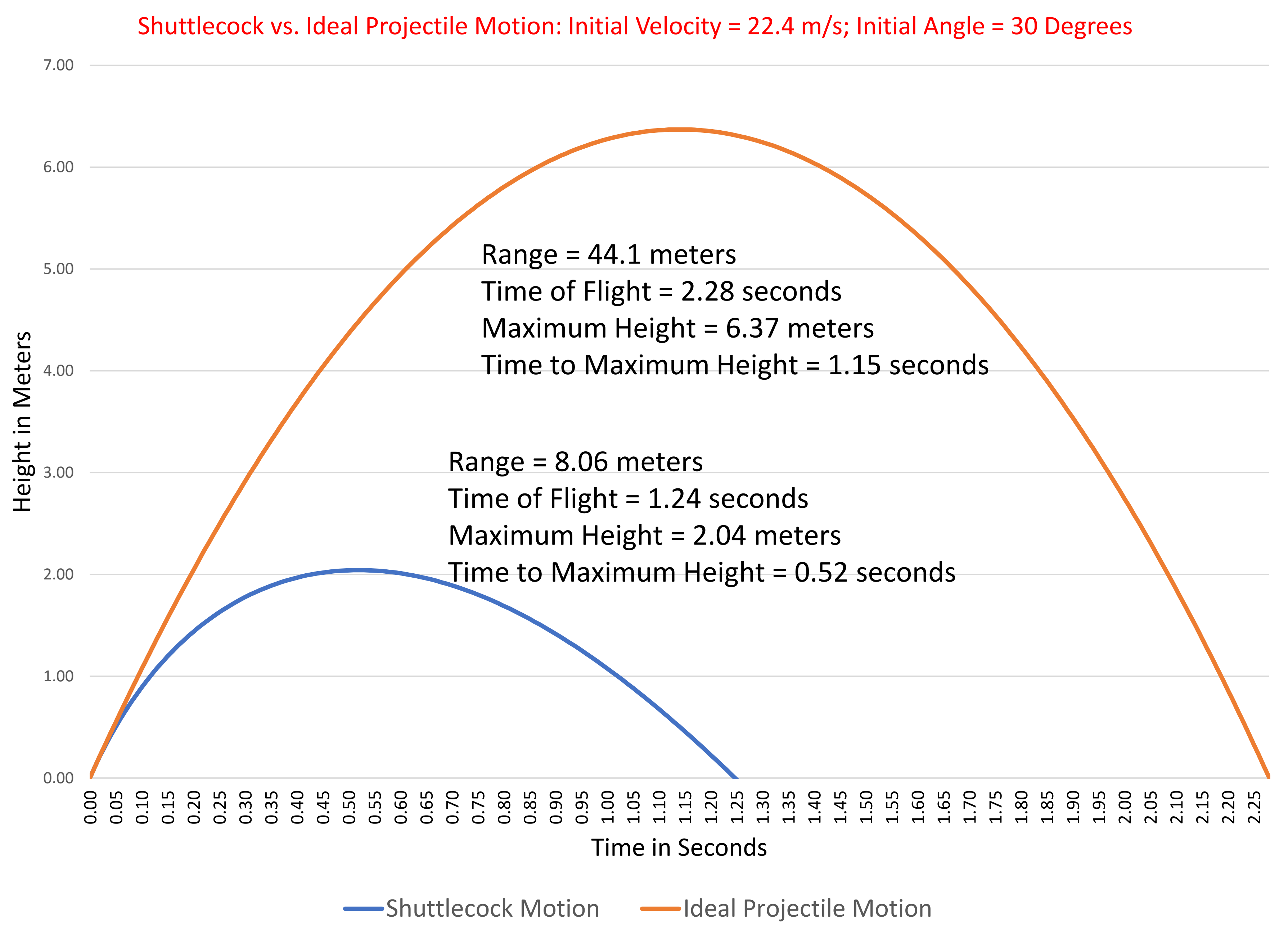
Bootstrap – Step II

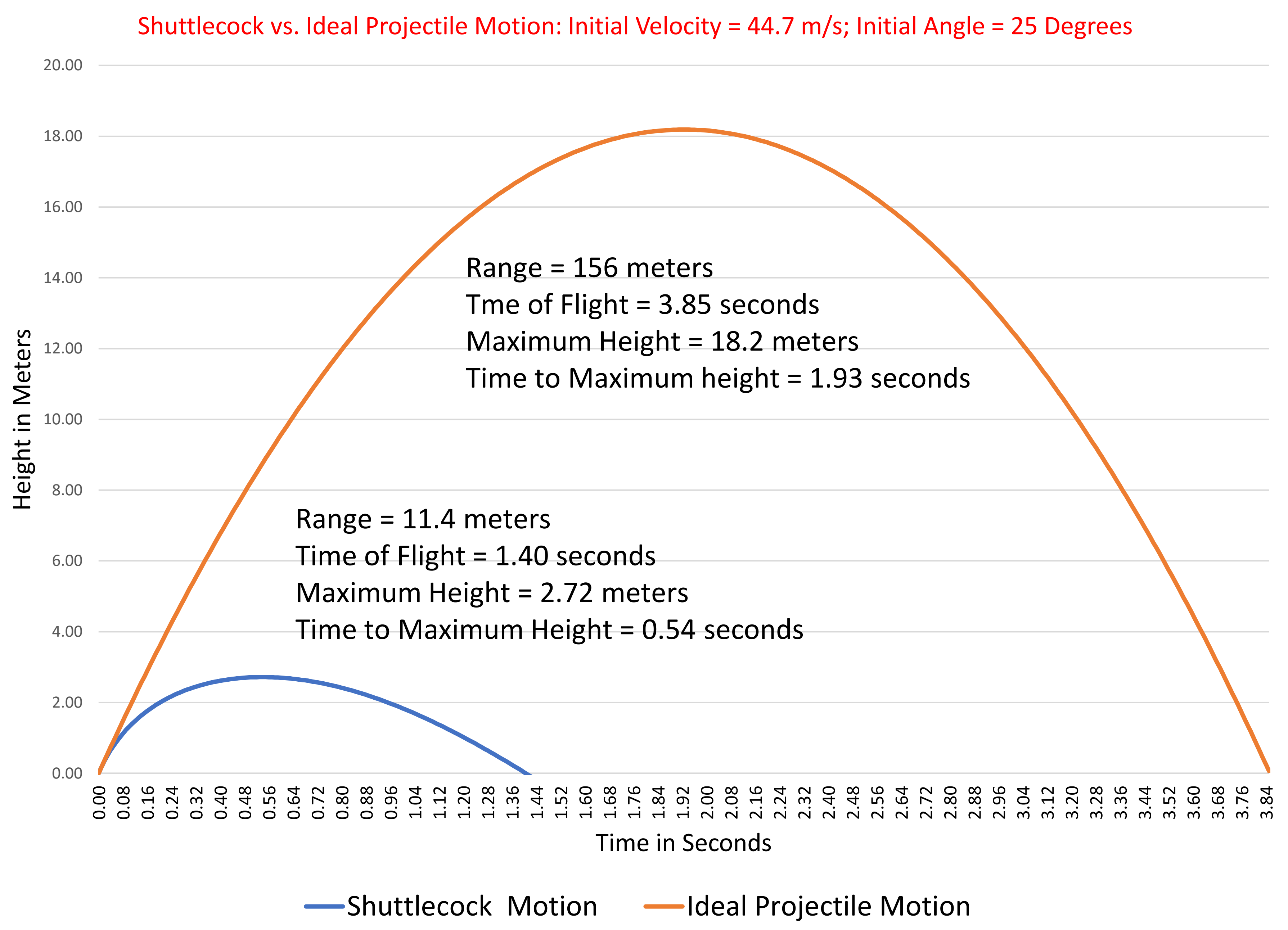
- The entries in the next (first) row of the spreadsheet will carry the following values:

  $0.01;\ 0.01;\ v_{x1} = v_{x0} + \Delta v_{x0};\ v_{y1} = v_{y0} + \Delta v_{y0};\ v_1;\ \Delta v_{x1};\ \Delta v_{y1}$
- Alongside the above columns, we will add columns with headings for x, y, Δx, and Δy.

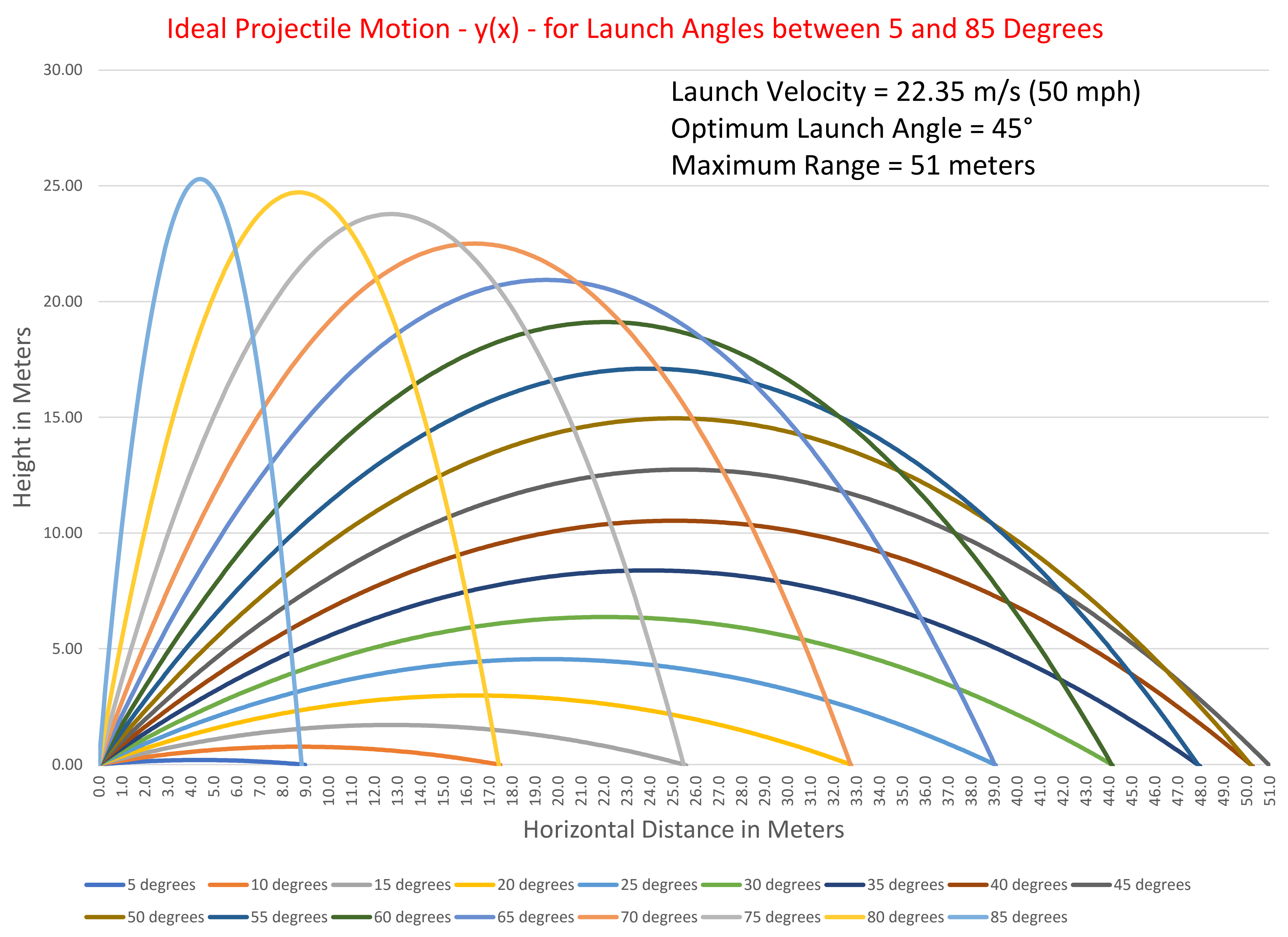
  Recall that by definition: $v_x = \Delta x/\Delta t$ and $v_y = \Delta y/\Delta t$

  This means that $\Delta x = v_x\Delta t$ and $\Delta y = v_y\Delta t$.
- And since we know the values of x_0 and y_o to be zero at $t = 0$, we can then calculate the values of x and y at time $t = 0 + \Delta t = \Delta t$: They are $x_1 = 0 + \Delta x_o$; and $y_1 = 0 + \Delta y_o$.
- For the next row, $x_2 = x_1 + \Delta x_1$, and $y_2 = y_1 + \Delta y_1$.
- Note: <u>Every new row in the spreadsheed is based on known values in the previous row</u>!
- And Excel makes it easy to generate the later rows by just building on the previous rows!

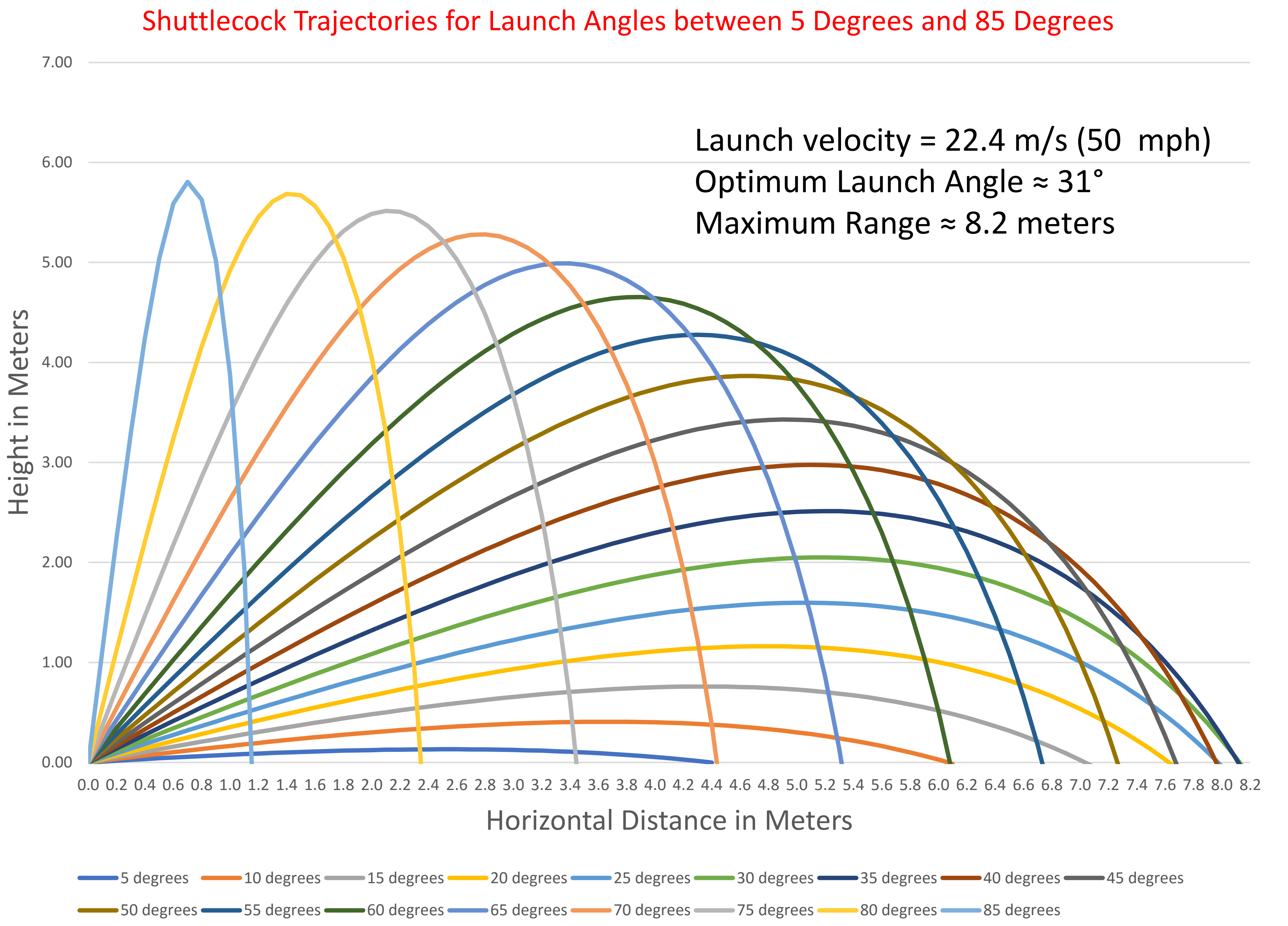

Shuttlecock vs. Ideal Projectile Motion: Initial Velocity = 22.4 m/s; Initial Angle = 30 Degrees
Height in Meters
Time in Seconds
Range = 44.1 meters
Time of Flight = 2.28 seconds
Maximum Height = 6.37 meters
Time to Maximum Height = 1.15 seconds
Range = 8.06 meters
Time of Flight = 1.24 seconds
Maximum Height = 2.04 meters
Time to Maximum Height = 0.52 seconds
Shuttlecock Motion
Ideal Projectile Motion



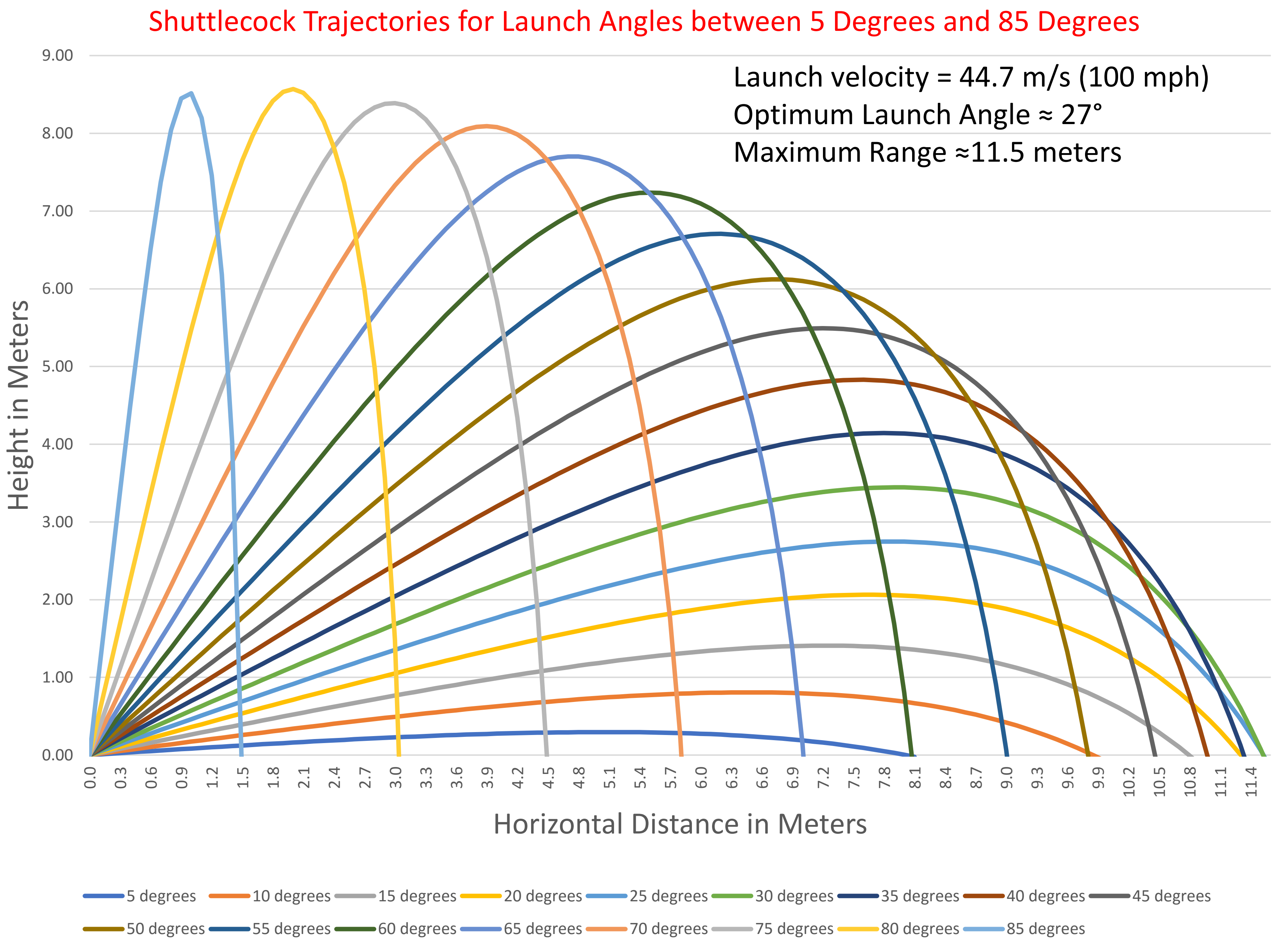
Shuttlecock vs. Ideal Projectile Motion: Initial Velocity = 44.7 m/s; Initial Angle = 25 Degrees
Height in Meters
20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
0.00
Range = 156 meters
Tme of Flight = 3.85 seconds
Maximum Height = 18.2 meters
Time to Maximum height = 1.93 seconds
Range = 11.4 meters
Time of Flight = 1.40 seconds
Maximum Height = 2.72 meters
Time to Maximum Height = 0.54 seconds
0.00 0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64 0.72 0.80 0.88 0.96 1.04 1.12 1.20 1.28 1.36 1.44 1.52 1.60 1.68 1.76 1.84 1.92 2.00 2.08 2.16 2.24 2.32 2.40 2.48 2.56 2.64 2.72 2.80 2.88 2.96 3.04 3.12 3.20 3.28 3.36 3.44 3.52 3.60 3.68 3.76 3.84
Time in Seconds
Shuttlecock Motion
Ideal Projectile Motion

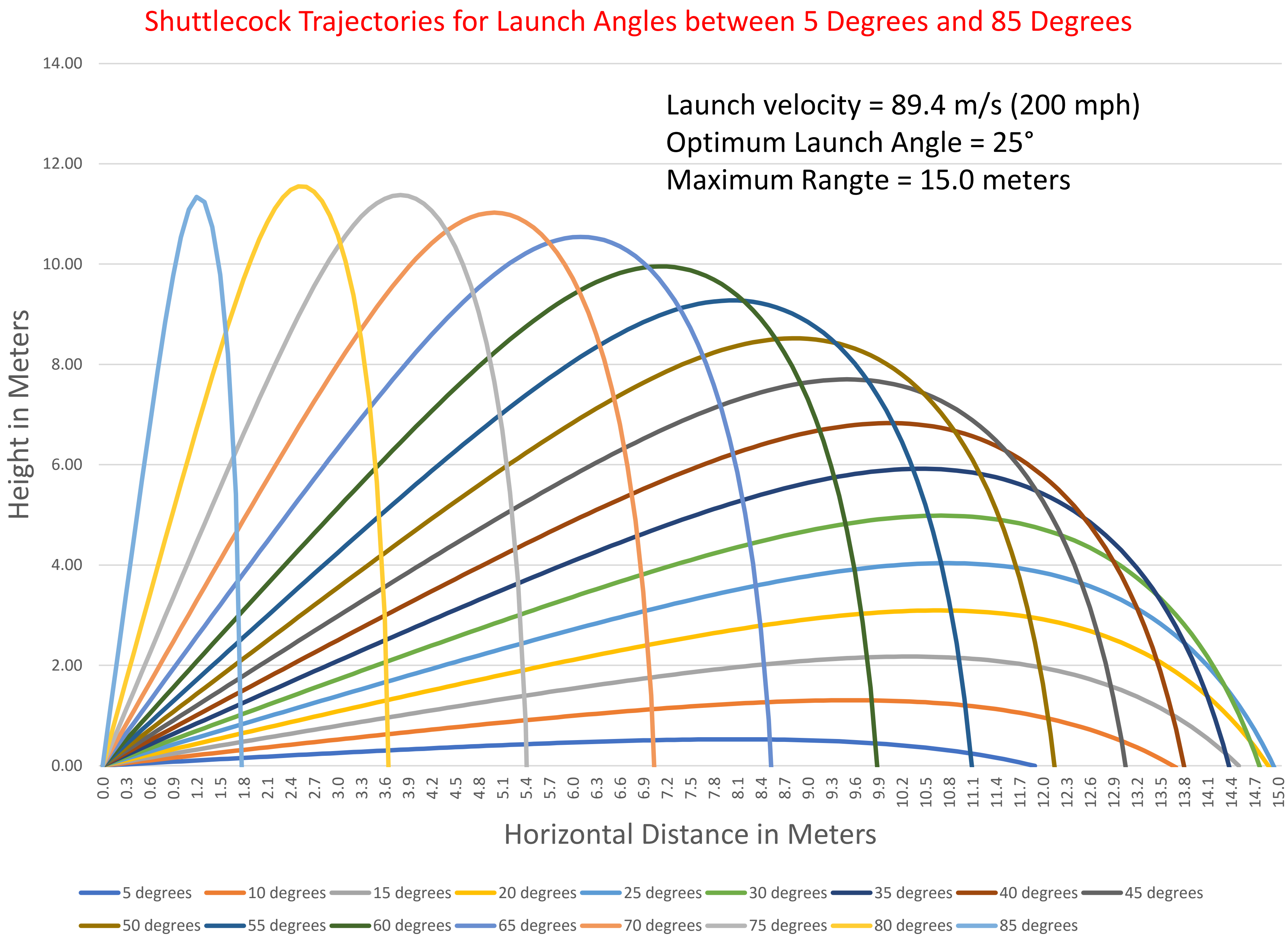


Ideal Projectile Motion - y(x) - for Launch Angles between 5 and 85 Degrees
Launch Velocity = 22.35 m/s (50 mph)
Optimum Launch Angle = 45°
Maximum Range = 51 meters
Height in Meters
Horizontal Distance in Meters
30.00
25.00
20.00
15.00
10.00
5.00
0.00
5 degrees
10 degrees
15 degrees
20 degrees
25 degrees
30 degrees
35 degrees
40 degrees
45 degrees
50 degrees
55 degrees
60 degrees
65 degrees
70 degrees
75 degrees
80 degrees
85 degrees



Shuttlecock Trajectories for Launch Angles between 5 Degrees and 85 Degrees
Launch velocity = 22.4 m/s (50 mph)
Optimum Launch Angle ≈ 31°
Maximum Range ≈ 8.2 meters
Height in Meters
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0 3.2 3.4 3.6 3.8 4.0 4.2 4.4 4.6 4.8 5.0 5.2 5.4 5.6 5.8 6.0 6.2 6.4 6.6 6.8 7.0 7.2 7.4 7.6 7.8 8.0 8.2
Horizontal Distance in Meters
5 degrees
10 degrees
15 degrees
20 degrees
25 degrees
30 degrees
35 degrees
40 degrees
45 degrees
50 degrees
55 degrees
60 degrees
65 degrees
70 degrees
75 degrees
80 degrees
85 degrees


Shuttlecock Trajectories for Launch Angles between 5 Degrees and 85 Degrees
Launch velocity = 44.7 m/s (100 mph)
Optimum Launch Angle ≈ 27°
Maximum Range ≈11.5 meters
Height in Meters
9.00
8.00
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
0.0 0.3 0.6 0.9 1.2 1.5 1.8 2.1 2.4 2.7 3.0 3.3 3.6 3.9 4.2 4.5 4.8 5.1 5.4 5.7 6.0 6.3 6.6 6.9 7.2 7.5 7.8 8.1 8.4 8.7 9.0 9.3 9.6 9.9 10.2 10.5 10.8 11.1 11.4
Horizontal Distance in Meters
5 degrees
10 degrees
15 degrees
20 degrees
25 degrees
30 degrees
35 degrees
40 degrees
45 degrees
50 degrees
55 degrees
60 degrees
65 degrees
70 degrees
75 degrees
80 degrees
85 degrees


Shuttlecock Trajectories for Launch Angles between 5 Degrees and 85 Degrees
Launch velocity = 89.4 m/s (200 mph)
Optimum Launch Angle = 25°
Maximum Rangte = 15.0 meters
Height in Meters
Horizontal Distance in Meters
0.00
2.00
4.00
6.00
8.00
10.00
12.00
14.00
0.0
0.3
0.6
0.9
1.2
1.5
1.8
2.1
2.4
2.7
3.0
3.3
3.6
3.9
4.2
4.5
4.8
5.1
5.4
5.7
6.0
6.3
6.6
6.9
7.2
7.5
7.8
8.1
8.4
8.7
9.0
9.3
9.6
9.9
10.2
10.5
10.8
11.1
11.4
11.7
12.0
12.3
12.6
12.9
13.2
13.5
13.8
14.1
14.4
14.7
15.0
5 degrees
10 degrees
15 degrees
20 degrees
25 degrees
30 degrees
35 degrees
40 degrees
45 degrees
50 degrees
55 degrees
60 degrees
65 degrees
70 degrees
75 degrees
80 degrees
85 degrees

"The Effects of Coefficient of Restitution (COR) Variations on Long Fly Balls"

- This analysis is based on the David T. Kagan article in *The Physics of Sports, pp.* 65-68. The following quote is from the abstract:
- "The coefficient of restitution of baseballs is required to be 0.546 ± 0.032. These variations affect launch velocity & ultimately the range of fly balls."
- It is well known that air resistance effects on baseballs are too large to ignore. That is, the trajectories of baseballs are not perfect examples of "ideal projectile motion."
- To predict the motion of baseballs subject to gravity and air resistance (but without spin) requires an analysis similar to the one we did for the shuttlecock. Assume an air resistance force proportional to v^2.

Terminal Velocity of the Baseball

- In 1959 scientist Lyman J. Briggs was the first to prove the curve ball was real and not an optical illusion, as many people had thought.
- Briggs' cited the following terminal velocity for the baseball:

$$v_T = 140 \text{ f/s} = 95.45 \text{ mph} = 42.67 \text{ m/s}.$$

- COR is measured experimentally by using an "air-cannon" to fire baseballs at a solid wall of "ash," the wood baseball bats have historically been made from.
- They measure initial speed v_i of the ball <u>before</u> it hits the wall, and the final speed v_f of the ball <u>after</u> it hits.

$$COR = v_f/v_i$$

The Meaning of the Term

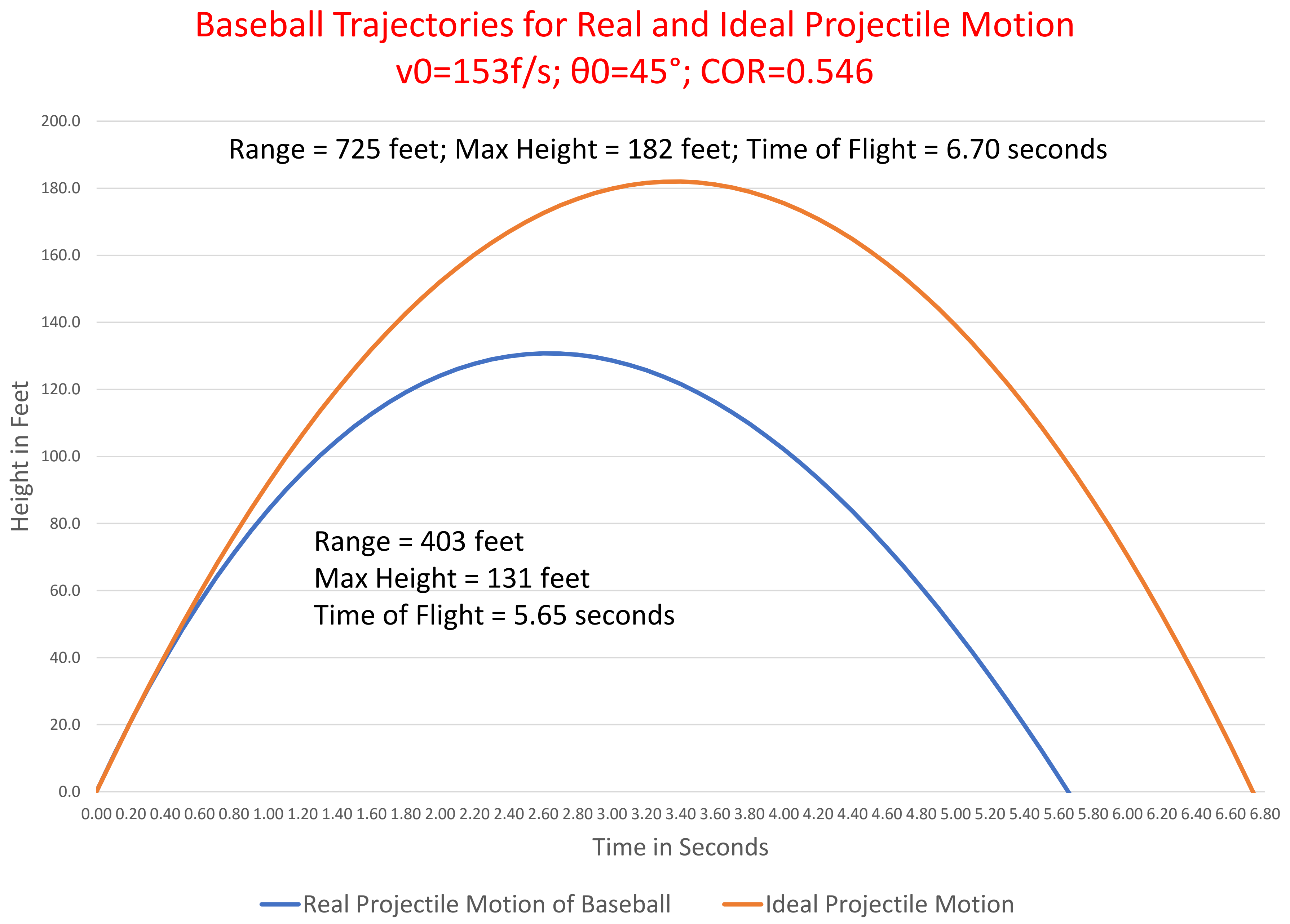
- A COR of 0.546 means the rebound velocity v_f is 54.6 percent of the incoming velocity.
- COR is a measure of how efficient the collision was in preserving speed and hence kinetic energy.
- By way of comparison, the COR of a tennis ball is 0.728.
- That MLB specifies that the COR for baseballs must fall in a range of 0.546 ± 0.032 introduces some ambiguity into just how "bouncy" any given baseball is permitted to be.
- At the low end of the range, the COR for a given ball could be as low as 0.546 – 0.032 = 0.514;
- At the high end of the range, the COR could be as high as 0.546 + 0.032 = 0.578.

The Size of COR Variations

- For the same incoming speed before a collision with the bat, the final speed after the collision with the bat could be either 51.4% of the incoming speed or 57.8% of the incoming speed.
- That is a percent difference defined by the ratio 57.8/51.4 = 1.125.
- The final speed of the ball on the high end of the COR scale could be 12.5% higher than the final speed of the ball on the low end.
- And if that higher “final speed” were coming off the bat of an MLB slugger, the difference in range could be very significant.

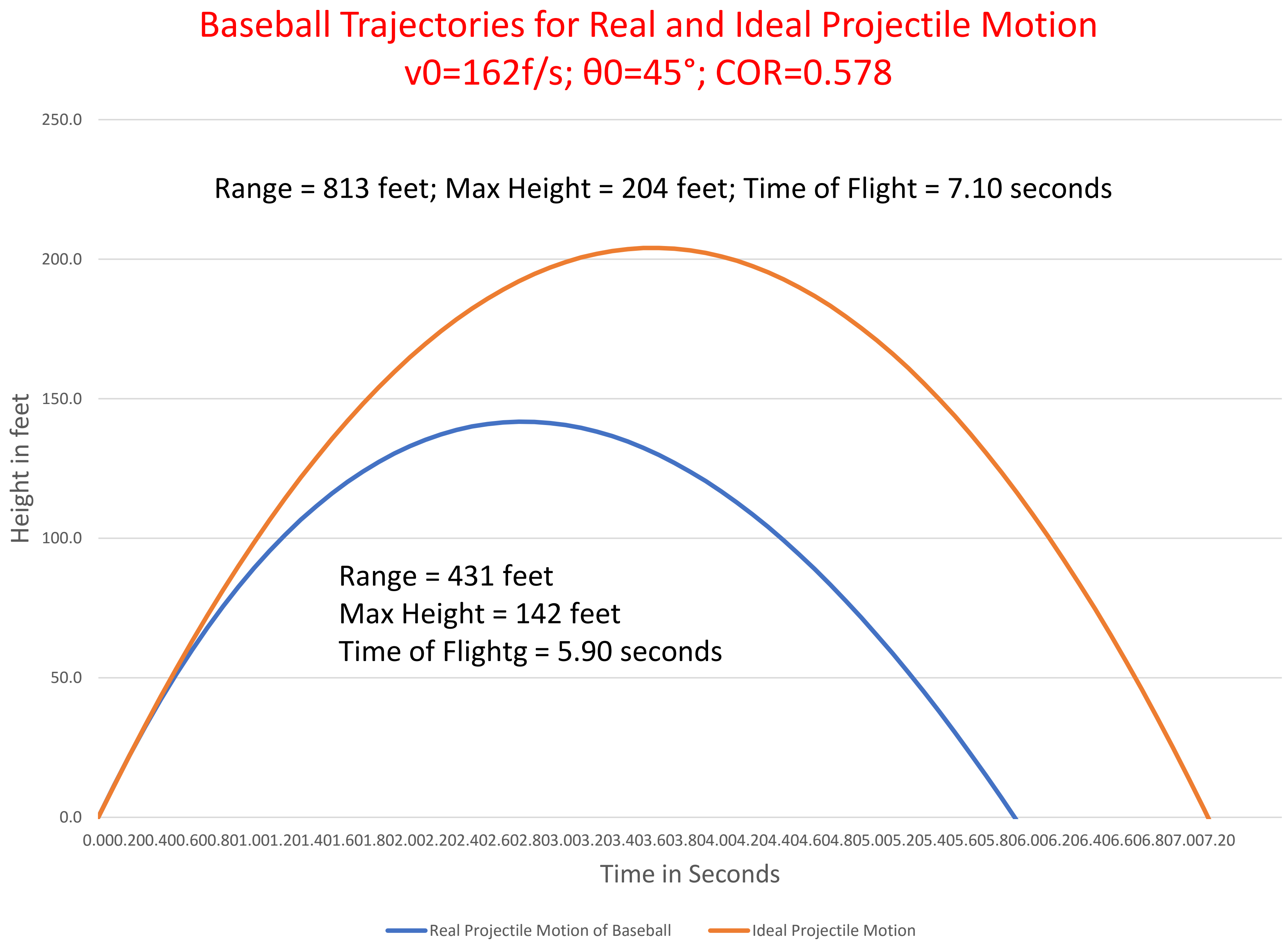
A Long Fly Ball to the Wall

- Assume that the same slugger hits the same pitch exactly the same way, first with a ball having a COR exactly equal to 0.546, right in the middle of the allowed range, and later with a COR on the high end.
- Using numerical integration with an Excel spreadsheet, I found one (of no doubt many) combinations of launch velocity and launch angle (v_0,θ_0) that yielded a fly ball near the 400 foot sign in center field in most ballparks.
- The first combination had launch velocity 153 f/s (104 mph) and a launch angle of 45°. Look at the next chart.


Baseball Trajectories for Real and Ideal Projectile Motion
v0=153f/s; θ0=45°; COR=0.546
Range = 725 feet; Max Height = 182 feet; Time of Flight = 6.70 seconds
Range = 403 feet
Max Height = 131 feet
Time of Flight = 5.65 seconds
Height in Feet
200.0
180.0
160.0
140.0
120.0
100.0
80.0
60.0
40.0
20.0
0.0
0.00 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00 2.20 2.40 2.60 2.80 3.00 3.20 3.40 3.60 3.80 4.00 4.20 4.40 4.60 4.80 5.00 5.20 5.40 5.60 5.80 6.00 6.20 6.40 6.60 6.80
Time in Seconds
Real Projectile Motion of Baseball
Ideal Projectile Motion

Range Variation with COR Variation

- The previous simulation of a long fly ball to the wall in the presence of air resistance yielded a horizontal range of 403 feet.
- The next simulation assumes a launch velocity at the high end of the COR range, which involves a final speed increase of 5.86percent (0.578/0.546 = 1.0586).
- That leads to a new launch velocity: 153f/s*(1.0586)=162f/s.
- See the next chart.


Baseball Trajectories for Real and Ideal Projectile Motion
v0=162f/s; θ0=45°; COR=0.578
Range = 813 feet; Max Height = 204 feet; Time of Flight = 7.10 seconds
Range = 431 feet
Max Height = 142 feet
Time of Flightg = 5.90 seconds
Height in feet
250.0
200.0
150.0
100.0
50.0
0.0
0.000.200.400.600.801.001.201.401.601.802.002.202.402.602.803.003.203.403.603.804.004.204.404.604.805.005.205.405.605.806.006.206.406.606.807.007.20
Time in Seconds
Real Projectile Motion of Baseball
Ideal Projectile Motion

The Bottom Line

- A 5.86% difference in launch speeds between the middle and top end of the COR values yields a 6.95% difference in fly ball range.
- The range for COR 0.546 was 403 feet, a ball the best center fielders would probably catch.
- The fly ball range for COR 0.578 was 431 feet, a sure home run in most ballparks.
- And we only used half the allowed COR range!

A Brief History of the "Curve Ball"

- This analysis is based on the Lyman J. Briggs article in *The Physics of Sports*, pp. 47-54. From the Abstract:
- "The effect of spin and speed on the lateral deflection (curve) of a baseball has been measured by dropping the ball while spinning about a vertical axis through the horizontal wind stream of a 6-ft tunnel."
- "For speeds up to 150 ft/sec and spins up to 1800 rpm, the lateral deflection was found to be proportional to the spin and to the square of the wind speed."
- "When applied to a pitched ball in play, the maximum expected curvature ranges from 10 to 17 inches, depending on the spin."
- Coincidentally, 17 inches—is the width of home plate!

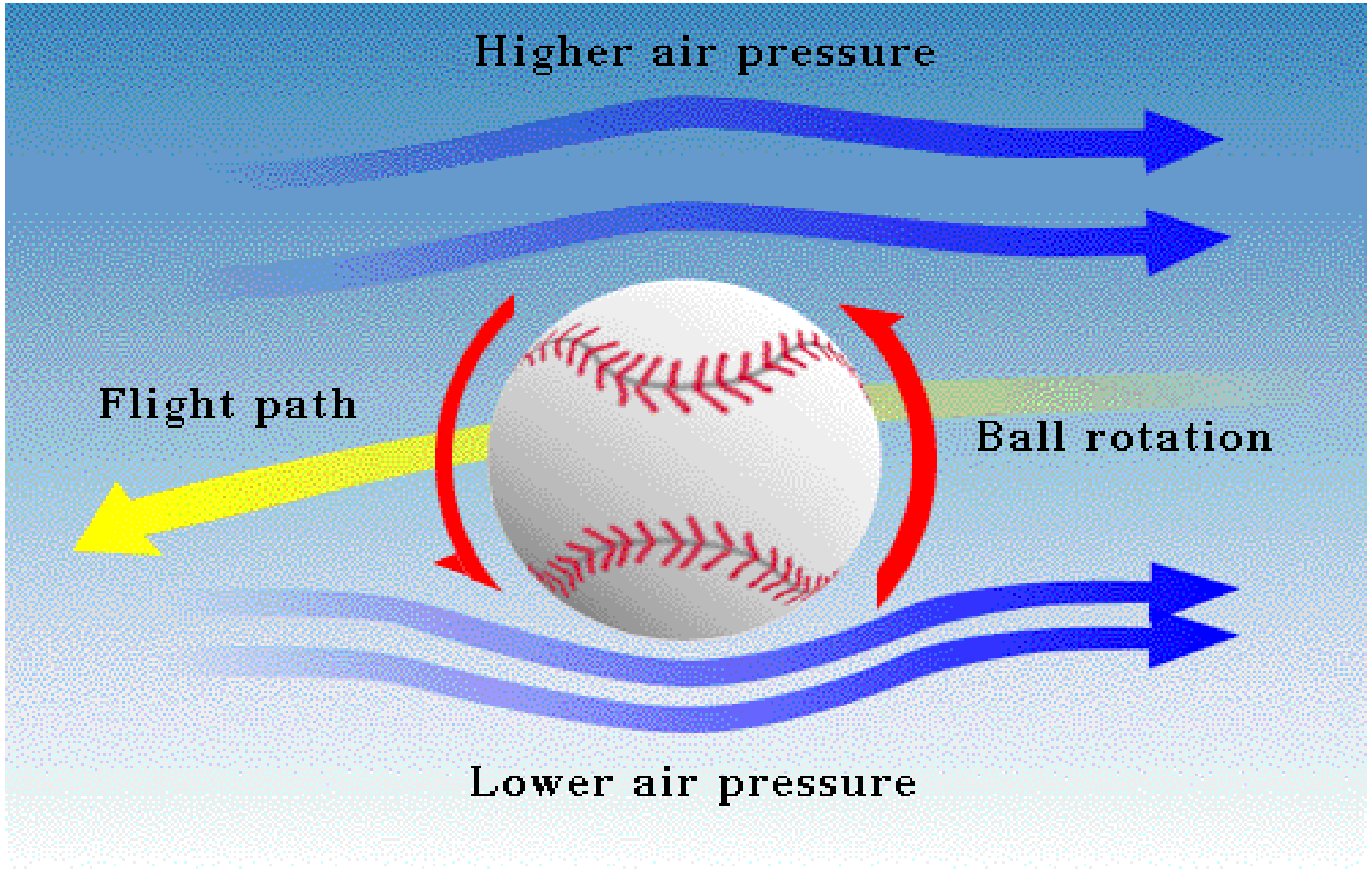
Dizzy Dean and the "Curve Ball"

- The history of the "curve ball" goes back to the 19th century. As of the 40s and 50s, major publications continued to call the curve ball an "optical illusion."
- Hall of Fame pitcher Dizzy Dean (1910-1974), who played between 1930 and 1947, said the following when someone said to him that the curve ball was an optical illusion:
- "Shucks, get behind a tree and I'll hit you with an optical illusion."

The Curve Ball is not an Optical Illusion!

- By 1959, however, the issue of whether the curve ball was real or not was settled once and for all by Lyman J. Briggs, then Director Emeritus of the National Bureau of Standards.
- A key result of Briggs' work (Table I) was this:
- With ω = 1800 rpm (30 rev/sec), and v = 125 ft/sec (85 mph), the measured deflection was 25.8 inches—more than two feet!

The Bernoulli Force on a Baseball



The Magnitude of the Spin Force

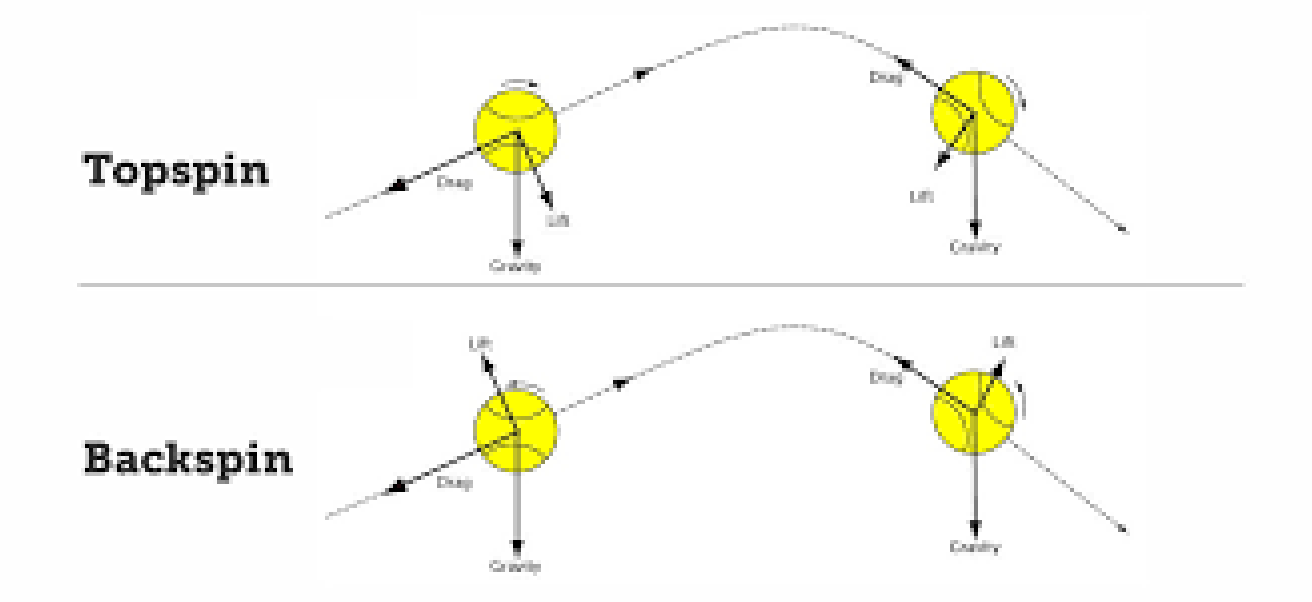
$$F_{spin} = c_L m \omega v^2$$

- c_L is a constant, and m is the mass of the baseball.
- Major league pitchers routinely put spin on the ball between 1200 and 1800 rpm, which is equivalent to 20 to 30 rev/second. A “knuckle ball” is thrown with spin from zero to 1 rev/second, at low speeds from 60 to 70 mph!
- The time of flight of a pitched ball between the mound and home plate is on the order of 0.5 second. So in that half-second time period, a ball rotating at 1800 rpm completes only 15 rotations.
- Yet the best hitters in major league baseball history have claimed they could detect the rate of spin and the axis of spin and hence know where the ball would likely end up when it crossed the plate! And this without ever having taken a physics course.

Motion of a Golf Ball with Backspin

- Based on experience with shuttlecocks and baseballs, one might expect that the projectile motion of a golf ball would obey the very same equations.
- But experiment shows that is not the case!
- For driven golf balls, both the resistive force and the lift force are proportional not to v^2 but to v!
- The air resistance force on a golf ball changes—not just in degree but in kind—depending on the speed of the ball. This change in character is known as the "drag crisis" of the golf ball.
- Below speeds of about 20 m/s (45 mph), the air resistance on a golf ball is proportional to v^2, the square of the ball's speed. But above 20 m/s, the air resistance on a golf ball is proportional to the first power of v.
- In what follows, we will therefore use the linear versions of the resistive forces and the lift forces.

Lift-Force Direction relative to Velocity Vector



The Equations of Motion

- We will assume a PGA type launch velocity of 200 ft/sec (136 mph). We will look at a range of launch angles to see which angles deliver the longest range for that particular launch velocity.
- We will assume that the spin imparted to the golf ball will be pure back spin. This will preclude any 3-dimensional motions; that is, no hooks, slices, draws or fades.
- For the x and y components of Newton's second law of motion for a golf ball with backspin we find:

$$-mcv_x - mkv\sin\theta = m(\Delta v_x/\Delta t), \qquad (4)$$

$$-mcv_y + mkv\cos\theta - mg = m(\Delta v_y/\Delta t). \qquad (5)$$

- The experimental values of the constants are: $c = 0.250s^{-1}$ and $k = 0.247s^{-1}$.

Linear Resistance and Linear Spin-Lift

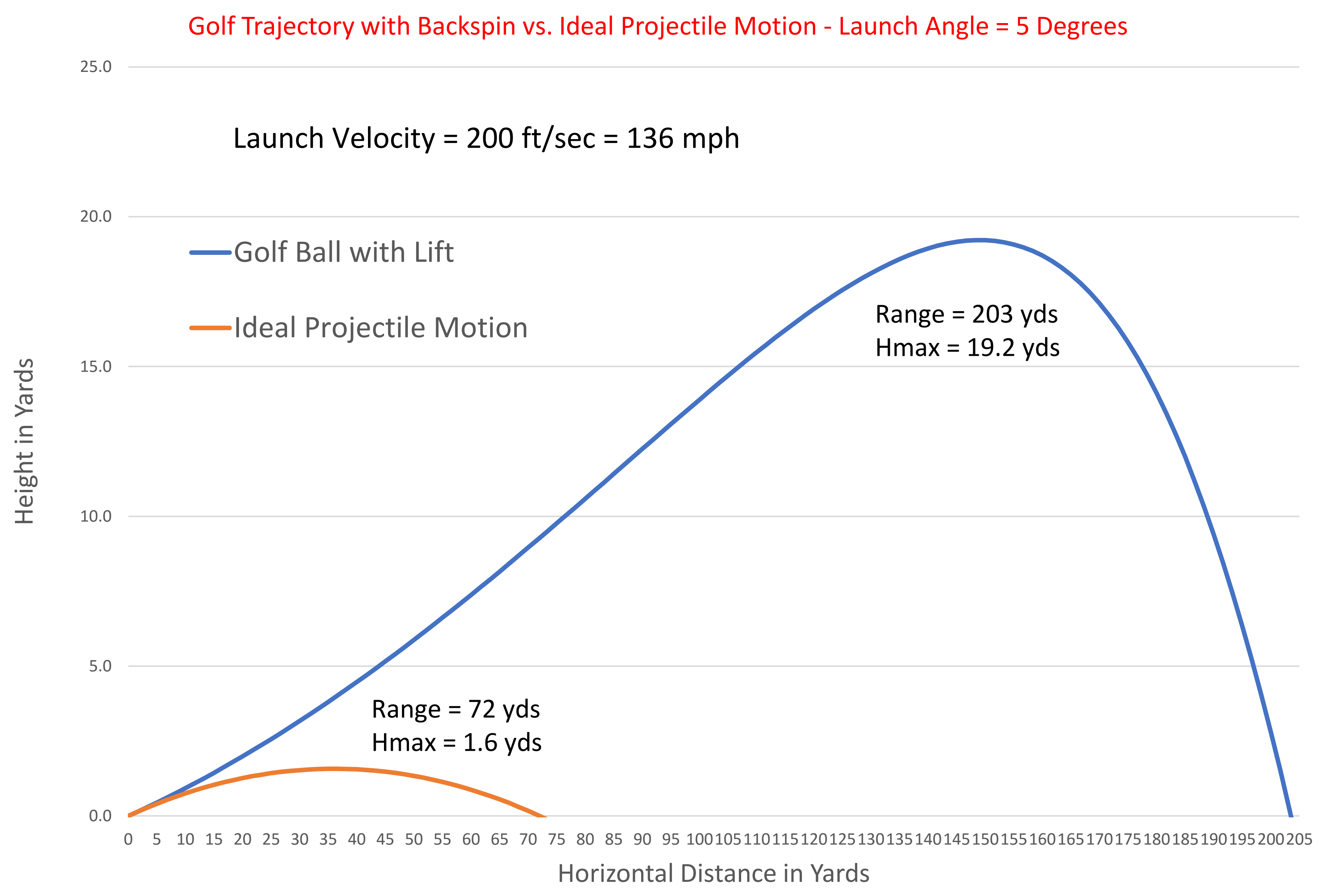
- By dividing out the mass terms on both sides, we obtain:

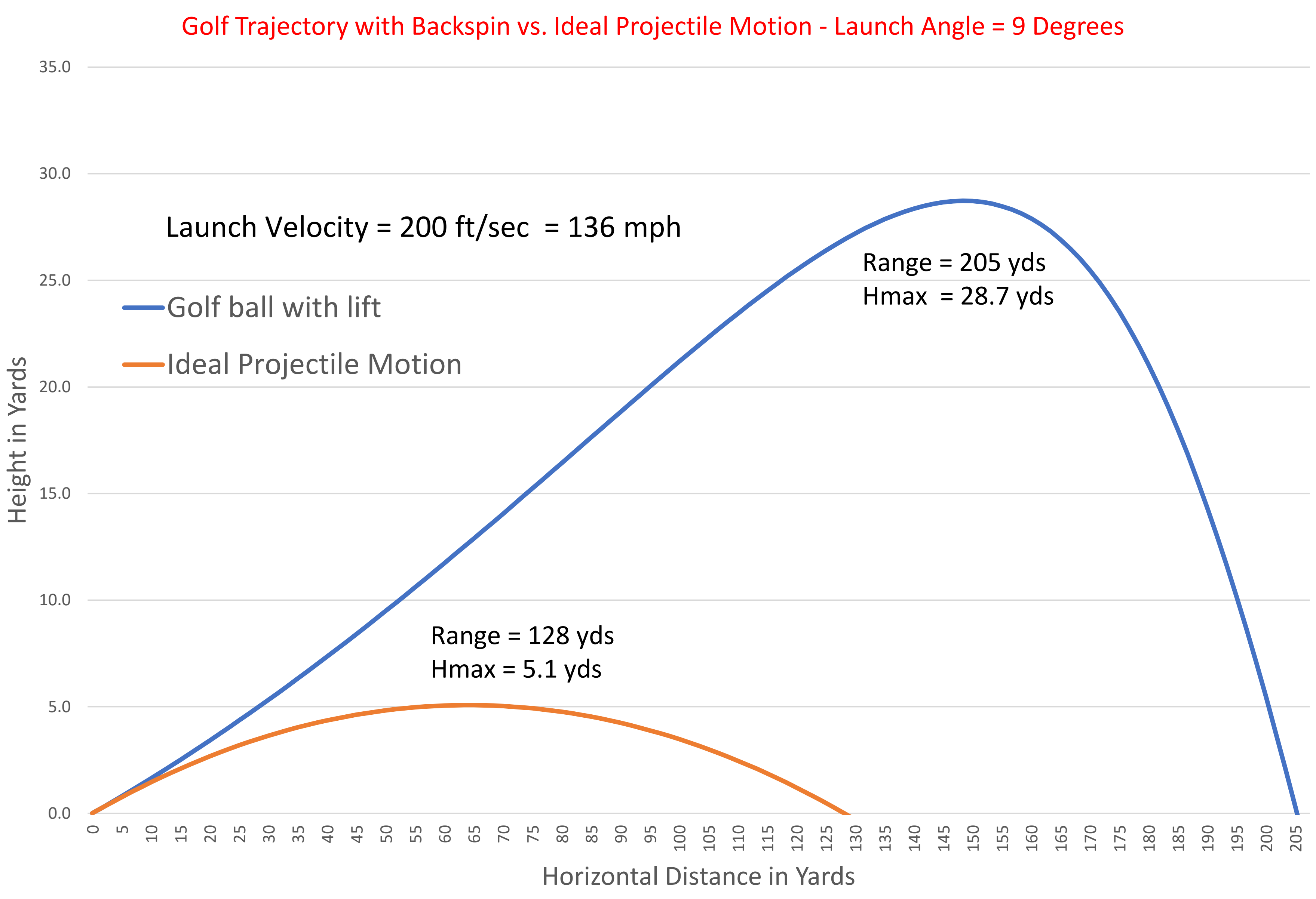
$$-cv_x - kv_y = (\Delta v_x/\Delta t), \qquad (6)$$

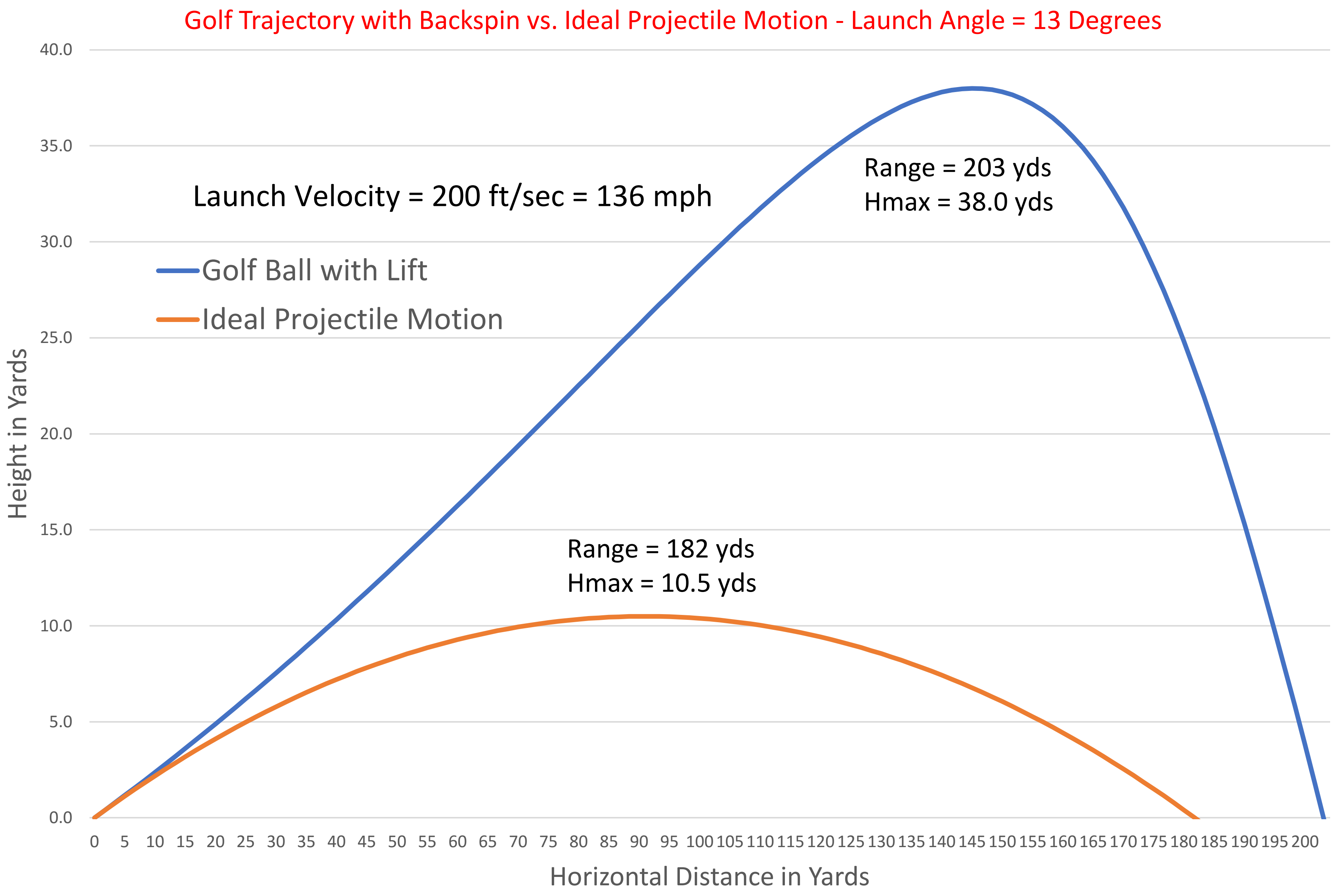
$$-cv_y + kv_x - g = (\Delta v_y/\Delta t), \qquad (7)$$

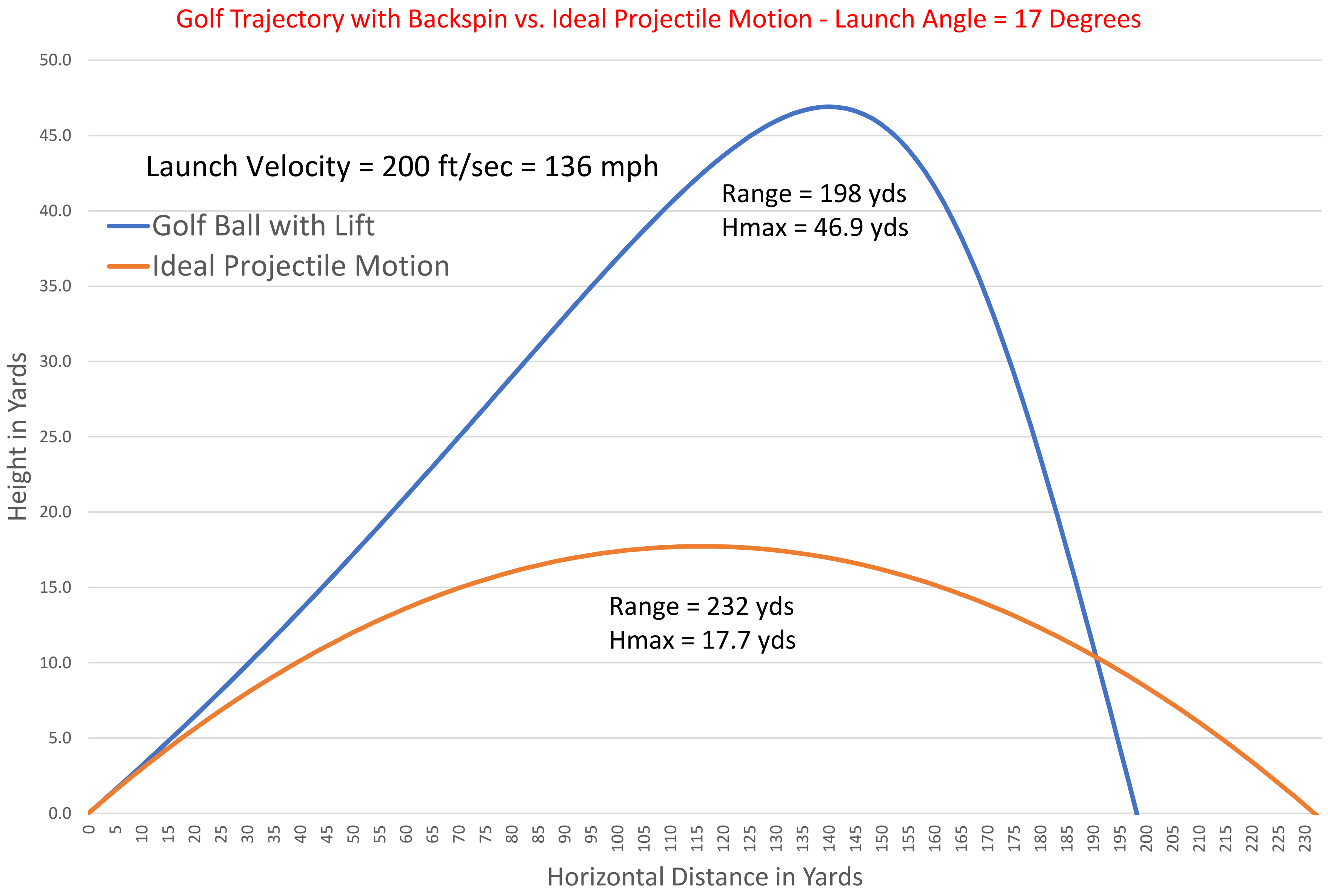
$$v^2 = v_x^{\,2} + v_y^{\,2}. \qquad (8)$$

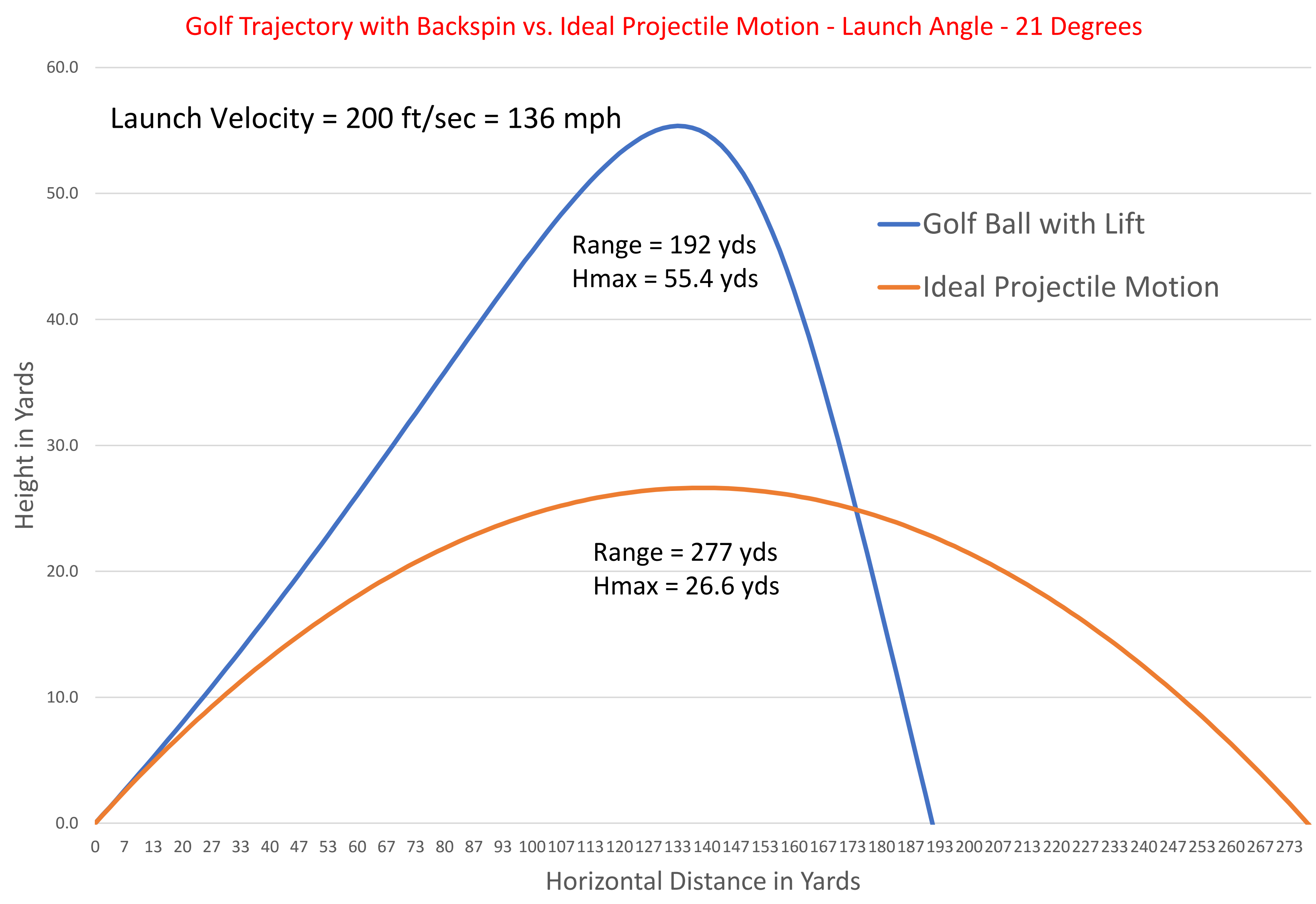
- These three equations were inserted into an Excel spreadsheet in which the numerical integration was conducted as we have done previously with coupled differential equations that cannot be solved exactly. We used a $\Delta t = 0.1$ second and the spreadsheets required 100 to 200 rows to give complete trajectories.

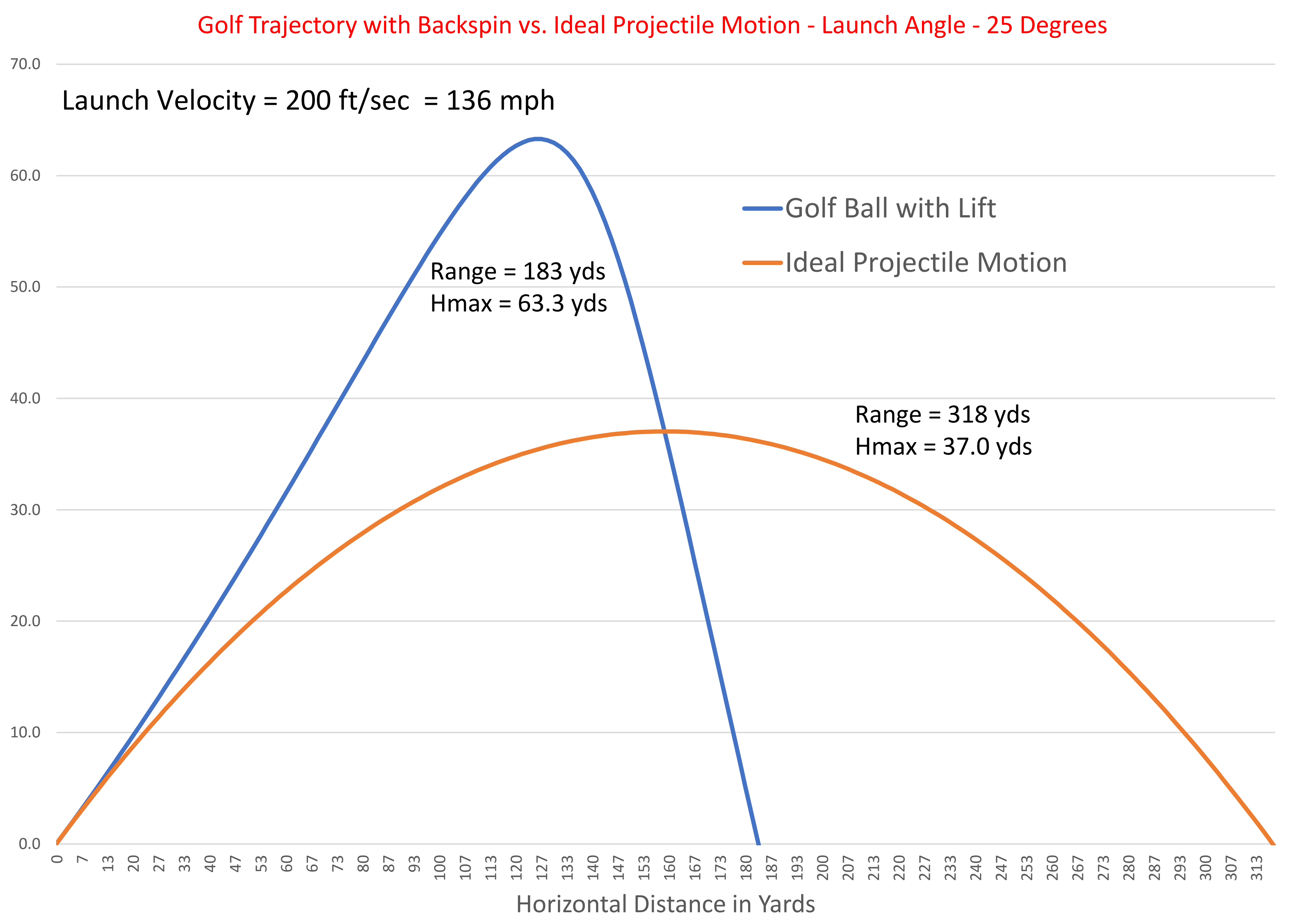

Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle = 5 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Golf Ball with Lift
Ideal Projectile Motion
Range = 203 yds
Hmax = 19.2 yds
Range = 72 yds
Hmax = 1.6 yds
Height in Yards
Horizontal Distance in Yards
25.0
20.0
15.0
10.0
5.0
0.0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205

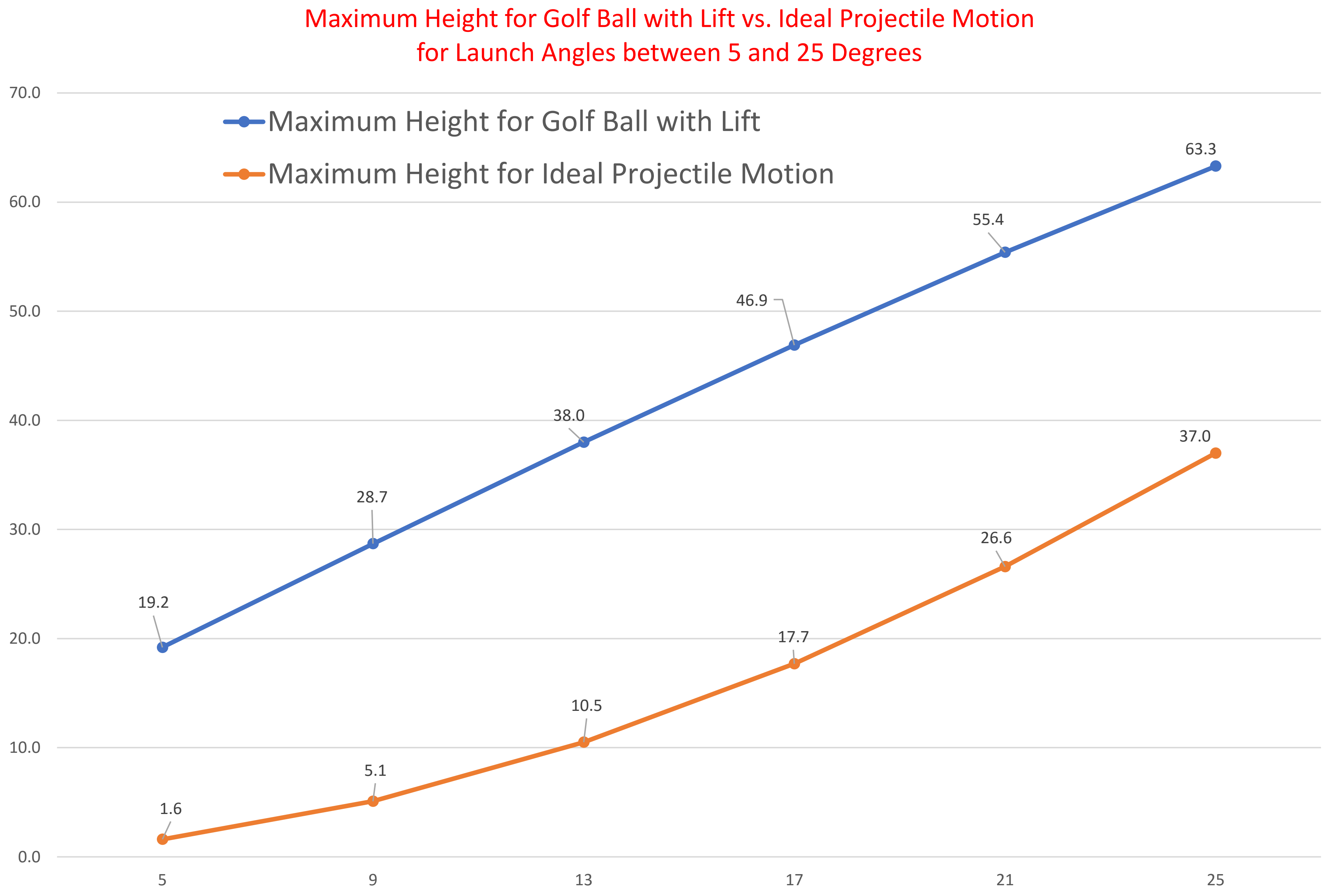

Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle = 9 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Golf ball with lift
Ideal Projectile Motion
Range = 205 yds
Hmax = 28.7 yds
Range = 128 yds
Hmax = 5.1 yds
Height in Yards
Horizontal Distance in Yards
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205


Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle = 13 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Golf Ball with Lift
Ideal Projectile Motion
Range = 203 yds
Hmax = 38.0 yds
Range = 182 yds
Hmax = 10.5 yds
Height in Yards
Horizontal Distance in Yards
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200

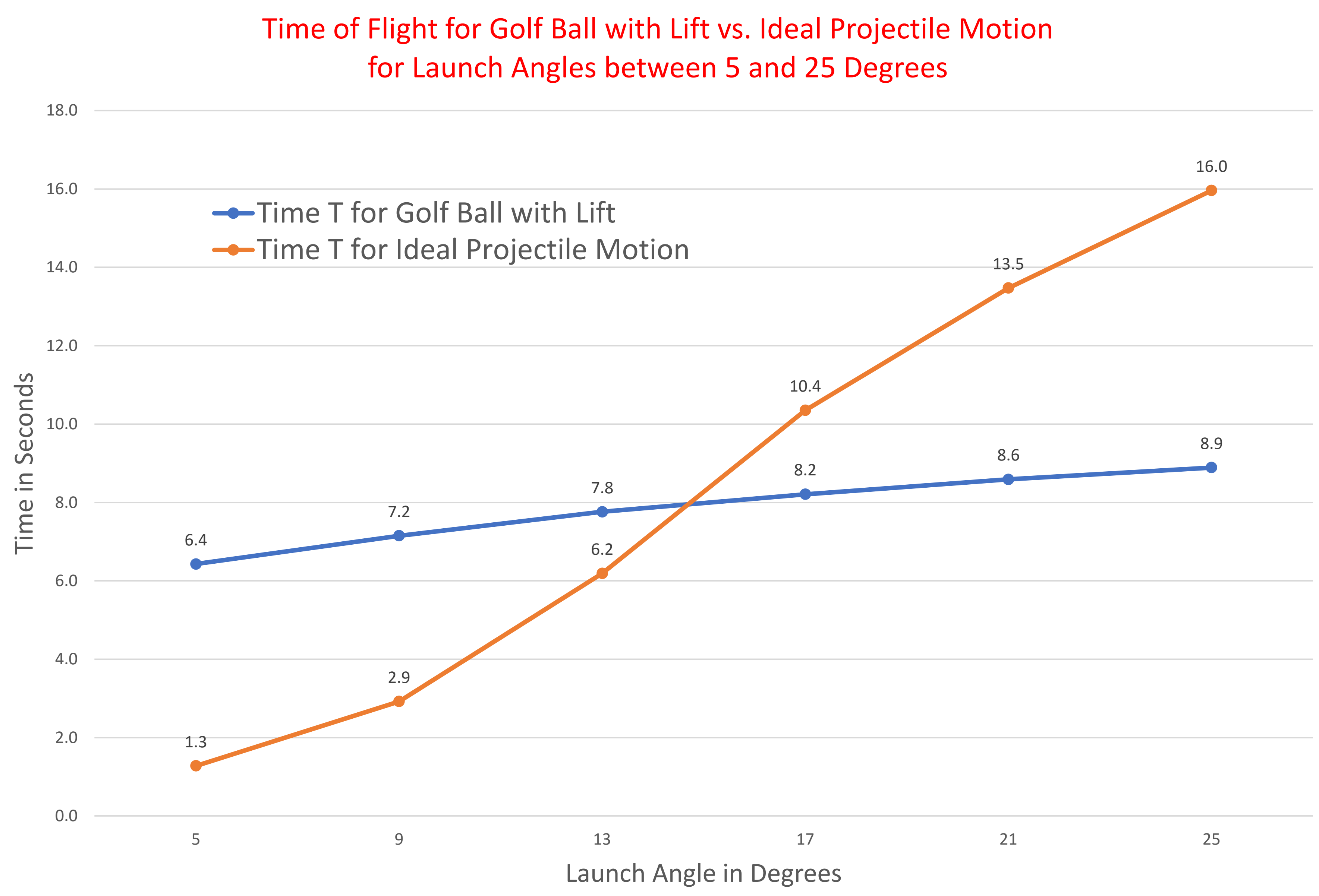

Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle = 17 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Golf Ball with Lift
Ideal Projectile Motion
Range = 198 yds
Hmax = 46.9 yds
Range = 232 yds
Hmax = 17.7 yds
Height in Yards
Horizontal Distance in Yards
50.0
45.0
40.0
35.0
30.0
25.0
20.0
15.0
10.0
5.0
0.0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200 205 210 215 220 225 230


Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle - 21 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Range = 192 yds
Hmax = 55.4 yds
Range = 277 yds
Hmax = 26.6 yds
Golf Ball with Lift
Ideal Projectile Motion
Height in Yards
Horizontal Distance in Yards
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0 7 13 20 27 33 40 47 53 60 67 73 80 87 93 100 107 113 120 127 133 140 147 153 160 167 173 180 187 193 200 207 213 220 227 233 240 247 253 260 267 273

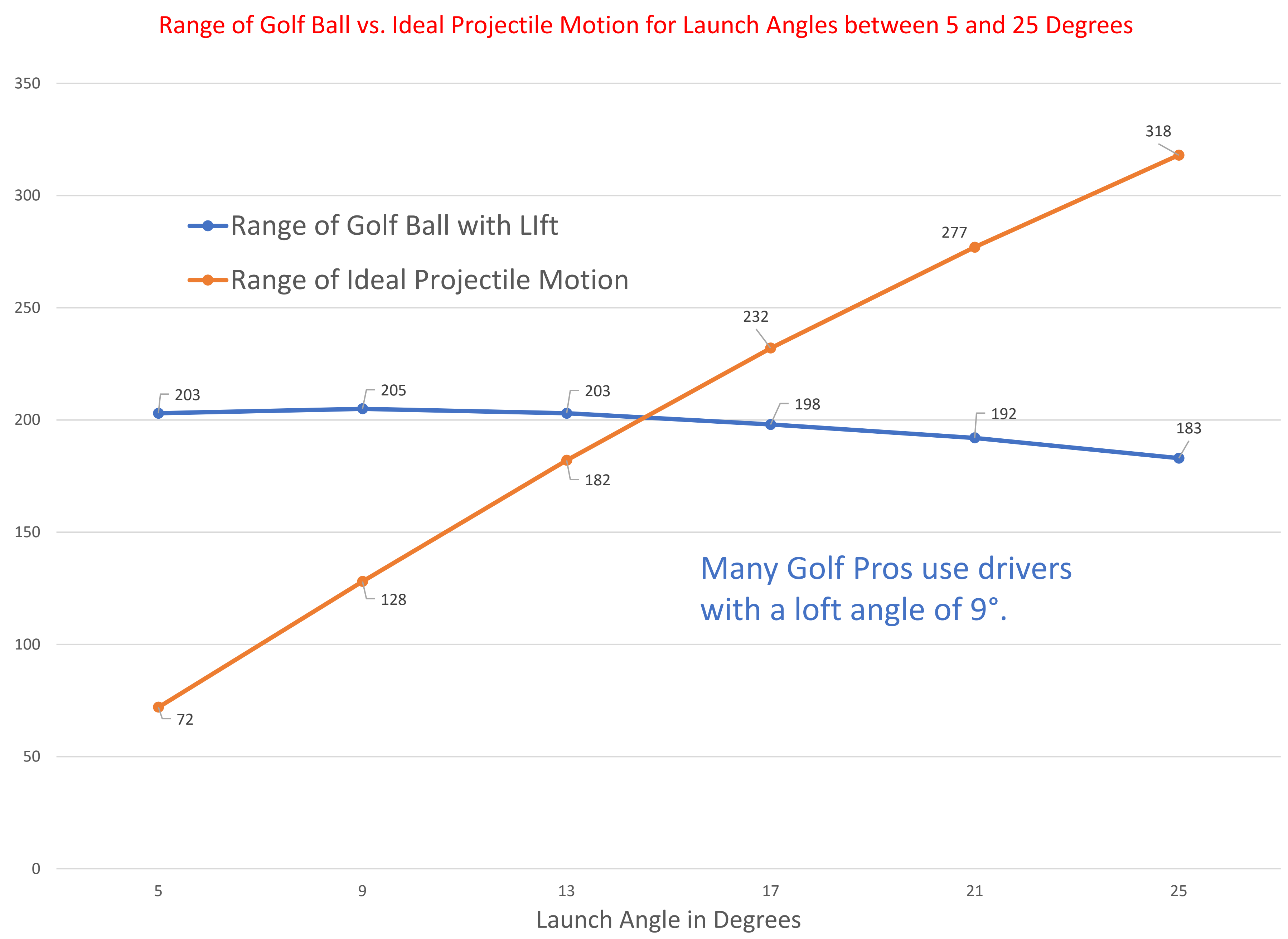

Golf Trajectory with Backspin vs. Ideal Projectile Motion - Launch Angle - 25 Degrees
Launch Velocity = 200 ft/sec = 136 mph
Golf Ball with Lift
Ideal Projectile Motion
Range = 183 yds
Hmax = 63.3 yds
Range = 318 yds
Hmax = 37.0 yds
Horizontal Distance in Yards
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
0 7 13 20 27 33 40 47 53 60 67 73 80 87 93 100 107 113 120 127 133 140 147 153 160 167 173 180 187 193 200 207 213 220 227 233 240 247 253 260 267 273 280 287 293 300 307 313



Maximum Height for Golf Ball with Lift vs. Ideal Projectile Motion
for Launch Angles between 5 and 25 Degrees
Maximum Height for Golf Ball with Lift
Maximum Height for Ideal Projectile Motion
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
5
9
13
17
21
25
19.2
28.7
38.0
46.9
55.4
63.3
1.6
5.1
10.5
17.7
26.6
37.0



Time of Flight for Golf Ball with Lift vs. Ideal Projectile Motion
for Launch Angles between 5 and 25 Degrees
Time T for Golf Ball with Lift
Time T for Ideal Projectile Motion
Time in Seconds
Launch Angle in Degrees
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0.0
5
9
13
17
21
25
6.4
7.2
7.8
8.2
8.6
8.9
1.3
2.9
6.2
10.4
13.5
16.0



Range of Golf Ball vs. Ideal Projectile Motion for Launch Angles between 5 and 25 Degrees
Range of Golf Ball with LIft
Range of Ideal Projectile Motion
350
300
250
200
150
100
50
0
5
9
13
17
21
25
Launch Angle in Degrees
203
205
203
198
192
183
72
128
182
232
277
318
Many Golf Pros use drivers
with a loft angle of 9°.

Thank You for your kind attention

- My Villanova email address is angelo.armenti@Villanova.edu.
- I welcome your questions and comments and will be happy to share some of my spreadsheets with you if you would like to see how they work and what they predict.
- All students (Arts Majors) in my MSE 2604 Physics of Sports course are required to do a project. One student (an excellent golfer) was able to model every club in his bag by starting with his known club head speed and the loft angle of every club.
- Stay well.   Check out “Nasty Pitches – Part 1” below.
- https://www.youtube.com/watch?v=Ps8yWie33Ek.