Application of the Fresnel Equations: Dielectric Cylinder

Reference:


Introduction:

A laser beam is transmitted into a dielectric glass cylinder and the polarization of the beam that emerges after a single internal reflection is studied as a function of the angle of incidence. The experimental results are compared with the predictions of Maxwell's Equations as expressed in the Fresnel Equations. By a general fit of the experimental data to the theoretical model a best estimate of the refractive index of the cylinder is obtained.

finally emerges at point c where its intensity is measured by a detector shown in Fig. 2.

The glass cylinder is placed on top of a sheet of polar graph paper that is not shown in the figure. The origin of the graph paper is at the location a and the laser beam is initially aligned with the 0-180 degree axis of the graph paper. The polar graph paper permits a convenient and accurate measurement of the angle of incidence $i$, the angle of refraction $r$ and the angle $\phi$. The angle of incidence is one half of the angle between the incident and reflected waves at a. The angle of refraction $r$ is most accurately obtained by first measuring the angle $\phi$. This angle is obtained by noting the location b where the transmitted beam strikes the cylinder's surface. From the figure it is seen that:

$$ r = i - \phi $$

(1)

From $i$ and $r$ the refractive index of the glass can be computed. In the following discussion the beam emerging at c will be referred to as the "scattered beam". The angle $\theta$ will be called the scattering angle for convenience but it may be worth noting that in other types of scattering the angle $\pi-\theta$ is often used as the scattering angle. It will be shown that as the angle of incidence $i$ is increased from 0 to $\pi/2$ the scattering angle increases to a maximum value and then decreases. Measurements are made of the polarization of the scattered beam with increasing angle of incidence. Finally the experimental polarizations are compared with the theoretical predictions. For large angles of incidence the

Scattering Geometry
Fig. 1

The scattering geometry used is depicted in Fig. 1. The laser beam first strikes the cylinder at location a. There the beam is both transmitted and reflected. The transmitted portion of the beam is subsequently reflected at b and then
phenomenon of "negative polarization" is examined.

Theory:

Maximum scattering angle:

The following relation between the scattering angle \( \theta \) and the angles \( i \) and \( r \) is easily obtained from Fig. 1:

\[
\theta = 4r - 2i \tag{2}
\]

The angles \( i \) and \( r \) are related by:

\[
\sin i = n \sin r \tag{3}
\]

where \( n \) is the refractive index of the cylinder. This equation will be used to obtain the refractive index.

It can be shown that the scattering angle has a maximum \( \theta_m \) that occurs for a value \( i_m \) of the angle of incidence. From Eqs. 2 and 3 it is found that the maximum scattering angle occurs for:

\[
\sin i_m = \left(\frac{4 - n^2}{3}\right)^{\frac{1}{2}} \tag{4}
\]

Eq.4 can be used to determine \( i_m \) once the index of refraction has been measured. Then Eqs. 3 and 2 can be solved for \( \theta_m \).

Polarization of the scattered beam:

The laser beam is assumed to be unpolarized. The plane of the page in Fig.1 contains the incident and scattered beams. This plane will be referred to as the plane of incidence. The Fresnel Equations, obtained by applying the boundary conditions required at the interface by Maxwell's Equations, are used to determine the intensity components in the scattered beam. It should be noted that the intensity rather than energy transmission and reflection coefficients must be used since only a portion of the scattered beam is sampled by the small detector. The Fresnel transmission and reflection coefficients for intensity are:

\[
T_{\text{par}} = \frac{n_i}{n_l} \left(\frac{2n_l \cos \theta_i}{n_i \cos \theta_i + n_l \cos \theta_i}\right)^2 \tag{5}
\]

\[
T_{\text{per}} = \frac{n_i}{n_l} \left(\frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_i \cos \theta_i}\right)^2 \tag{6}
\]

\[
R_{\text{par}} = \left(\frac{n_i \cos \theta_i - n_i \cos \theta_i}{n_i \cos \theta_i + n_i \cos \theta_i}\right)^2 \tag{7}
\]

\[
R_{\text{per}} = \left(\frac{n_i \cos \theta_i - n_i \cos \theta_i}{n_i \cos \theta_i + n_i \cos \theta_i}\right)^2 \tag{8}
\]

where \( n_i \) and \( n_l \) are the refractive indices of the material containing the incident and transmitted waves respectively and \( \theta_i \) and \( \theta_l \) are the angles of incidence and refraction respectively. The angles shown in Fig. 1 must be interpreted correctly. For example, the angle \( r \) at location \( e \) is the angle of incidence while \( i \) is the angle of refraction. If the laser beam is assumed to have equal intensity components \( I_0 \) in the directions perpendicular and parallel to the plane of incidence then the intensity of the scattered components will be:

\[
I_{\text{par}} = T_{\text{par}}^a R_{\text{par}}^b T_{\text{par}}^c I_0 \tag{9}
\]

\[
I_{\text{per}} = T_{\text{per}}^a R_{\text{per}}^b T_{\text{per}}^c I_0 \tag{10}
\]

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where the superscripts a,b and c refer to the locations marked in Fig. 1. The theoretical value of the polarization will be defined in the usual manner as:

\[ P_{\text{theory}} = \frac{I_{\text{per}} - I_{\text{par}}}{I_{\text{per}} + I_{\text{par}}} \]  

(11)

and the unknown factor \( I_0 \) of Eqs. 9 and 10 cancels.

Experimental:

Detector-amplifier:

The detector consists of a silicon photo voltaic cell connected with short leads to a pre amplifier circuit

\[ \lambda \rightarrow \text{Photo diode} \rightarrow \text{Pre amp} \rightarrow \text{To amp circuit} \]

Current to Voltage  
Pre Amplifier

Fig. 2

that converts the current into a proportional voltage. The preamplifier minimizes noise and produces a sufficiently large voltage such that any noise induced in the leads connecting it to the main amplifier circuit will be insignificant. This second amplifier is used to control the GAIN and ZERO and is shown in Fig. 3. The buffer amplifiers provide a high input and low output impedance. The Zero Adjust amplifier is used to set the background level. The switch in the Select Gain amplifier feedback loop is used to choose the feedback resistor for a high or low gain.

![Signal Amplifiers](Fig. 3)

The amplifier is saturated when its output level goes beyond 13 volts. If this occurs you must reduce the amplification level or the laser intensity.

Polaroid filter:

A mask is provided that consists of two polaroid films having transmission axes that are perpendicular and parallel to the plane of incidence. These filters will be referred to as perpendicular and parallel polarizers respectively and will be placed in the laser beam as shown in Fig. 1.

Data collection:

For each new angle of incidence you should adjust the amplifier gain to give a reasonably large signal since the polarization involves the ratio of intensities and is independent of the amplifier gain. However do not adjust the gain while measuring the vertical and

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horizontal components described below for a particular angle of incidence.

- Align the laser beam so that it is traveling directly along the 0-180 axis of the polar graph paper.
- Place the glass cylinder in position such that the laser strikes its surface directly above the origin of the polar graph paper and the angle between the incoming and reflected beams at point a is exactly 15 degrees.
- Place the vertical polaroid in the laser path (before it reaches the glass cylinder) and record the intensity of the transmitted beam that emerges at point c.
- Place the horizontal polaroid in the laser path (before it reaches the glass cylinder) and record the intensity of the transmitted beam that emerges at point c.
- Record the background intensity by blocking the laser beam. For each intensity measurement subtract the background value.
- Record the angle 2\(i\) between the incident angle and the reflected wave at point a. Also record the angle \(\phi\) between the 0-180 axis and the line connecting points a and b in Fig. 1.
- Repeat these measurements for the angle 2\(i\) ranging from 15 degrees to 170 degrees in 5 degree increments.

Analysis:

(a) Experimental polarization: For each intensity subtract the appropriate background intensities to obtain \(I_{\text{per}}\) and \(I_{\text{par}}\). Using these values compute the experimental polarization:

\[
P_{\text{exp}} = \frac{I_{\text{per}} - I_{\text{par}}}{I_{\text{per}} + I_{\text{par}}}\]  

(12)

(b) Determination of the refractive index by a general fitting method: It is possible to obtain a value for the refractive index by varying this parameter and searching for the minimum in the sum of the squares of the deviations between the observed and computed polarizations. This can be accomplished by writing your own computer program or by using the curve fit option of SigmaPlot. In order to use the curve fit option enter an initial guess for the refractive index as a parameter and enter the angles of incidence and the experimental values of the polarization as variables. In the equation region you will need to solve for the angle \(r\) using Eq. 3 and enter Eqs. 5,6,7,8,9,10 and 11 to obtain the calculated value for the polarization. Finally fit the calculated values of the polarizations to the experimental values. You can define functions for the transmission and reflection coefficients. In this way obtain the best estimate for the index of refraction, \(n\). Be sure to save this fit since it will be loaded later (for plotting purposes) into SigmaPlot as a transformation with only minor editing.

(c) Determination of the refractive index from Snell's Law: The index of refraction can be obtained from Eq. 3 using the measured values of the angles of incidence and refraction. However, unreliable values could be obtained when very small or very large angles of incidence are used. Why? Using only the angles of incidence in the range from 20 to 60 degrees compute the average index of refraction and its standard deviation.

(d) Polarization versus angle of incidence plots: Load the curve fit files

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into the transform option of SigmaPlot and edit out the unnecessary statements from the curve fit option. Generate separate theoretical polarization data for the angle of incidence in the range 0 to 90 degrees in increments of 0.2 degrees for both of the refractive indices obtained in (a) and (b). Plot a graph of P_{Exp} versus the angle of incidence, i. On the same graph plot the two theoretical polarizations curves. Compare the experimental and theoretical curves.

(e) Polarization versus the scattering angle: Plot the experimental values of the polarization as a function of the scattering angle, θ, rather than the angle i. The angle θ could be obtained from Eq. 2 using the measured values of r and i. However since this equation involves the difference between the angles i and r rather large fluctuations in θ are to be expected. For this reason it is better to use the measured value of i and the mean refractive index to compute the angle r from Eq. 3. Then use Eq. 2 to obtain the scattering angle θ. Estimate the maximum scattering angle from this graph. Using the average value of the refractive index, n, in Eq. 4 compute the theoretical value of i_m. Use this value to compute the corresponding maximum scattering angle from Eqs. 3 and 2. Compare the observed and theoretical values of the maximum scattering angles. Derive Eq. 4.

(f) "Negative polarization": If you have made careful measurements for large angles of incidence you may have been successful in observing "negative polarization" which of course means that the plane of polarization has changed by 90 degrees. This phenomenon is not widely known but was noticed in measurements of the polarization of EM waves reflected from the moon's surface at certain viewing angles.