different bands, we should add a symbol, say $n$, to the $C_i$'s to serve as a band index: $C_n(k + G)$. Thus the Bloch function for a state of wavevector $k$ in the band $n$ can be written as

$$\psi_{nk} = \exp(i k \cdot r) a_{nk}(r) = \sum_{g} C_{n}(k + G) \exp(i(k + G) \cdot r) .$$

**Periodic Zone Scheme**

We can repeat a given Brillouin zone periodically through all of wavevector space. To repeat a zone, we translate the zone by a reciprocal lattice vector. If we can translate a band from other zones into the first zone, we can translate a band in the first zone into every other zone. In this scheme the energy $\epsilon_k$ of a band is a periodic function in the reciprocal lattice:

$$\epsilon_k = \epsilon_{k+G} .$$

Here $\epsilon_{k+G}$ is understood to refer to the same energy band as $\epsilon_k$. 

Figure 4 Three energy bands of a linear lattice plotted in (a) the extended (Brillouin), (b) reduced, and (c) periodic zone schemes.
Figure 5 (a) Construction in $k$ space of the first three Brillouin zones of a square lattice. The three shortest forms of the reciprocal lattice vectors are indicated as $G_1$, $G_2$, and $G_3$. The lines drawn are the perpendicular bisectors of these $G_i$s. (b) On constructing all lines equivalent by symmetry to the three lines in (a) we obtain the regions in $k$ space which form the first three Brillouin zones. The numbers denote the zone to which the regions belong; the numbers here are ordered according to the length of the vector $G$ involved in the construction of the outer boundary of the region.

Figure 6 Brillouin zones of a square lattice in two dimensions. The circle shown is a surface of constant energy for free electrons; it will be the Fermi surface for some particular value of the electron concentration. The total area of the filled region in $k$ space depends only on the electron concentration and is independent of the interaction of the electrons with the lattice. The shape of the Fermi surface depends on the lattice interaction, and the shape will not be an exact circle in an actual lattice. The labels within the sections of the second and third zones refer to Fig. 7.

Figure 7 Mapping of the first, second, and third Brillouin zones in the reduced zone scheme. The sections of the second zone in Fig. 6 are put together into a square by translation through an appropriate reciprocal lattice vector. A different $G$ is needed for each piece of a zone.
Fold over extended BZ's into 1st BZ by cropping Fermi surface along with it.

Figure 8 The free electron Fermi surface of Fig. 6, as viewed in the reduced zone scheme. The shaded areas represent occupied electron states. Parts of the Fermi surface fall in the second, third, and fourth zones. The fourth zone is not shown. The first zone is entirely occupied.

Figure 9 The Fermi surface in the third zone as drawn in the periodic zone scheme. The figure was constructed by repeating the third zone of Fig. 8.

Brillouin zone (Fig. 7). Other reciprocal lattice vectors will shift the triangles $2_a, 2_b, 2_c$ to other parts of the first zone, completing the mapping of the second zone into the reduced zone scheme. The parts of the Fermi surface falling in the second zone are now connected, as shown in Fig. 8.

A third zone is assembled into a square in Fig. 8, but the parts of the Fermi surface still appear disconnected. When we look at it in the periodic zone scheme (Fig. 9), the Fermi surface forms a lattice of rosettes.

**Nearly Free Electrons**

How do we go from Fermi surfaces for free electrons to Fermi surfaces for nearly free electrons? We can make approximate constructions freehand by the use of four facts:

- The interaction of the electron with the periodic potential of the crystal creates energy gaps at the zone boundaries.
- Almost always the Fermi surface will intersect zone boundaries perpendicularly.

Figure 10 Qualitative surface of Fig. 8. At 2nd zone the energy increases toward the unshaded electron-like, whereas on

- The crystal potential
- The total wave electron core interaction.

We cannot make we expect the changed as show Freehand surfaces are us procedure credited mined, and a concentration at least one sp within at least and similarly fi

We said er interactions bet
they are connected by a reciprocal lattice vector. Such an orbit is called an **open orbit**. Open orbits have an important effect on the magnetoresistance.

Vacant orbitals near the top of an otherwise filled band give rise to hole-like orbits, as in Figs. 13 and 14. A view of a possible energy surface in three dimensions is given in Fig. 15.
Summary of this material:

- 3rd band: electron-like states
- 2nd band: hole-like states
- 1st band: totally full

Electro-lattice interaction will change shape of hole-like and electron-like states by deforming Fermi sphere.