1. It is expected that the melting point of a crystal should be linearly proportional to the cohesive energy, among other things. Unfortunately, relatively few “clean” systems (in which those “other things” are not at play, and answers are easy to calculate) are rare. Crystals of the noble gases are one such exception, which is why we spend so much time talking about them. Is this linear relationship between melting point and cohesive energy true? Plot the data (Table 4, Kittel chapter 3) to find out.

2. We may determine the ground-state crystal structure of a material by computing the total cohesive energy $U_{\text{tot}}$ for various structures and picking the one with the lowest energy. The lattice sums for BCC and FCC are:

   FCC: \[ \sum p_{ij}^{12} = 12.1319 \quad \sum p_{ij}^6 = 14.4519 \]

   BCC: \[ \sum p_{ij}^{12} = 9.11418 \quad \sum p_{ij}^6 = 12.2519 \]

Here, $p_{ij} = R_{ij}/R_0$, where $R_{ij}$ is the distance between the $i^{\text{th}}$ and $j^{\text{th}}$ atoms, and $R_0$ is the nearest neighbor distance.

Determine which structure should be the ground state (this result should be true for all the noble gas crystals). Note that the cohesive energy must be calculated in two steps: (1) you must find the equilibrium value of $R_0/\sigma$ by setting $dU_{\text{tot}}/dR = 0$ (once for each structure; Kittel demonstrates that $R_0/\sigma$ for FCC is 1.09, but we don’t know this yet for BCC), and (2) then you must compute $U_{\text{tot}}$ for each structure using the lattice sums above and the $R_0/\sigma$ value you calculated. Note that you don’t need to calculate the actual binding energy, just determine which one is the biggest.

3. a. All of the binding energy values you calculated in Problem 2 should be negative. What does that mean?

   b. Kittel points out that the real, experimental values of $R_0/\sigma$ for the noble gas crystals are slightly different from what you calculated above. In particular, the experimental values are slightly larger than what you calculate from evaluation of the lattice sums, and more so for the lighter noble gases. Why is this the case?

4. a. Iron has a Young’s Modulus of 211 GPa. If I have an iron cylinder 1 cm in radius and 1 m long, what is its “Hooke’s Law” k?

   b. Iron has a shear modulus of only 82 GPa. If I were to build a structure out of iron, is it stronger with respect to tension/compression, or to shear stresses?
5. The speed of sound in a material is defined as

\[ c = \sqrt{\frac{B}{\rho}} \]

where \( B \) is the bulk modulus, defined as

\[ B = -V \frac{dP}{dV} \]

and \( \rho \) is the density.

(a) What is the bulk modulus of an ideal gas, held at constant temperature?

(b) Compute the speed of sound in air (which we can assume to be an ideal gas), at standard temperature and pressure. (Is this number familiar?)

(c) The bulk modulus of steel is 160 GPa, and its density is about 7.9 g/cm\(^3\). What is the speed of sound in steel?

(Observation: in this problem you have related bulk mechanical properties, such as Hooke’s Law-type measurements of force and deflection, to the speed of propagation of sound waves through the material. We will expand on this idea in the next chapter!)